GROUP STRUCTURING IN ELEMENTARY SCHOOL MATHEMATICS: PEER-BASED OR ACADEMIC-BASED

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Abstract

This quasi-experimental pilot study examined how structuring of groups affects academic progress in math for grade 3 students (*N* = 71) from School X in Surrey, British Columbia. Participants completed the Vancouver Island Numeracy Assessment (INA) as a pre-test, followed by an intervention (group structuring: academic-based and peer-based). After 16 weeks, all participants, completed the INA a second time as a post-test to determine if the structuring of groups impacted mathematical abilities.

When mathematical performance was considered, a main effect was revealed: pre-test (M = 33.55, SD = 8.27, p < .001); post-test (M = 37.61, SD = -6.58, p < .001). No other main effects or interactions were observed between groups, or when sex and age were considered. Despite a small sample size and no significant results between groups, the study sheds initial ideas on the way in which groups are structured and paves the way for further inquiry.

Keywords: *Ability Grouping, Tracking, Collaboration, Mathematical Abilities*

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Alphabetical List of Acronyms and Abbreviations

ANOVA	Analysis of Variance
BC	British Columbia
BCTF	British Columbia Teacher Federation
EF	Executive Functioning
IEP	Individualized Education Plan
LP	Learning Plan
MKO	More Knowledgeable Other
NCTM	National Council of Teachers of Mathematics
REC/B	Research Ethics Committee/Board
SES	Socio-Economic Status
SPSS	Statistical Packages for Social Sciences
TIMSS	Trends in International Mathematics and Science Study
INA	Vancouver Island Numeracy Assessment *Note. The Vancouver Island Network has titled their resource as "Diagnostic Math Assessment (DMA)" – however, it is referred to as INA in their research*
ZPD	Zone of Proximal Development

Group Structuring in Elementary School Mathematics: Peer-based or Academic-based

Chapter 1: Introduction

Math is an essential skill for children to develop as it provides competency in the areas of number understanding, problem-solving, measurement, and the understanding of shapes and spatial awareness (Shanley et al., 2017). Historically, skills required for the workforce were focused on simple routines. However, those requirements have transitioned and instead many jobs require the use of problem-solving skills such as constructing, explaining, manipulating, and predicting to reach an end product (English et al., 2013). As our workforce is constantly evolving, our education system must ensure that students are adequately provided with the required skills, many of which are developed through the practice of mathematics (English et al., 2013). As math is considered an essential skill, it is important to determine the best approaches for teaching math in the classroom to ensure optimal academic progress and success.

In British Columbia (BC), curriculum development and assessment are an ongoing process that is guided by research and global trends (BC Curriculum, 2020). All participants in this study were grade three students; and therefore, the mathematical concept discussed will relate to this age group. The current Mathematics Curriculum Competencies for grade three require students to communicate mathematical thinking, use mathematical vocabulary to contribute to discussions, and connect mathematical concepts to previous content; as well as, other areas of personal interest (BC Curriculum, 2020). As educators, these components thus highlight the importance of recognizing how our current teaching practices allow for students to progress in these specific areas.

One method that is commonly used in an elementary setting is whole group teaching, where teachers guide students through new concepts by using a lecture-style approach, followed by an opportunity to use the new skills they have learned (Schulman, 1986). However, given that our current BC Curriculum requires certain skills (e.g., communication), teachers must ensure they are also providing students with opportunities to grow and develop such competencies, in addition to other skills.

It has been identified that when students begin a negative trajectory in math in early elementary years (K-3), they continue to follow the same path, resulting in deficiencies in mathematical skills in high school (Clements, 2011). When students begin to fall behind their peers in mathematics, it follows that they seemingly would become easily discouraged and accordingly, their math abilities could deteriorate further. With this in mind, it is imperative for educators to find methods that support both the academic and emotional needs of their students.

One instructional method that accounts for both a student's academic and emotional needs, is the use of social learning (Smith, 1996). Social learning theory, proposed by Albert Bandura (1977), identified that learning can occur through the process of observation and social interaction. One way this can occur is through collaborative work. As social learning can take on many forms, this paper will focus on the use of collaboration in small groups to explore the impact of social learning.

In the classroom, collaborative exercises can include working with one partner, working with a small group, or working with the whole class. However, the way in which the groups are structured can also vary as collaborative work can occur based on interest, academic level, or personal choice. Therefore, this pilot study will explore the use of two different methods for structuring groups: *peer-based* and *academic-based.* As the term "peer-based groups" is not commonly used in education, for the purpose of this study, a peer-based group will be determined by participants' self-reported peer relationships. "Academic-based" groups will be determined by a student's academic success, as determined by a pre-test. Grouping students based on their academic abilities is observed in education (Simon et al., 2007), however dividing students based on their academic ability or achievement is often referred to as ability grouping (Steenbergen-Hu et al., 2016) or tracking (Betts, 2011).

Background and Problem Statement

My original interest in the topic of this study came from previous observations, during 2018, when I put students in groups for language arts. In hopes that students would not realize that they had been divided by abilities, I labeled them by colour. However, two weeks into working with students based on abilities, one student said, "Miss Poon – why do I have to be in the *dumb* group? – I'm always in the *dumb* group and we never get to read the fun books." Knowing that after only two weeks, an 8-year-old student not only recognized that he was with students who were considered weaker, but that this was also something he continually faced, made me question how we as teachers group students. Often, I thought that I was helping students because I was able to meet them at their academic level and alter work to fit their needs; however, in doing this, I realized that I had already placed my own preconceived ideas on what they would achieve. As soon as we finished that book, I changed my method and allowed students to book two books they were interested in reading out of a selection I had chosen. This time, I did not worry about whether a book was too hard or too easy for a student, but instead just let them read what they wanted based on interest (ie. animals, sports, mystery, etc) while ensuring that books were at an appropriate grade level. While doing this, I noticed that when students were grouped based on their interest, their engagement increased, social skills (i.e., ability to communicate, respond respectfully, etc.) developed, and communication increased amongst them and their peers.

Based on these observations, I wondered how grade three students' academic performance could be affected when the structure of the group was reconsidered. I considered our current approach to mathematics in BC secondary schools where it has been determined that students should be grouped by their abilities, often referred to as tracks (Simon et al., 2007). For example, grade 10 students are required to pick a mathematical pathway between either Apprenticeship and Workplace Mathematics or Foundations of Mathematics and Pre-calculus (North Vancouver School District, n.d.), which has been observed to have a direct influence on their post-secondary options. Therefore, the desire of this pilot study was to determine whether grouping by ability, as observed in high schools, can benefit elementary schools mathematical performance.

When students complete their work early, I often ask if they would like to help another student. As a teacher, one method I frequently use involves partnering students as a form of peer-tutoring. Doing so, allows students to receive additional feedback when I am not accessible, while developing social skills through asking questions, explaining solutions, and interacting with peers. Since I have observed the benefits of group work and how secondary schools in the province construct their math programs, this raises questions regarding the best method for structuring at an elementary level.

The earliest study on ability grouping was published in 1922 (Seashore, 1922). Since then, numerous studies have been done in the subjects of both language arts and mathematics across various countries, where data has shown both positive and negative effects on ability grouping (Sahlberg, 2011). Ability grouping, which, for the purpose of this study, will be determined as a grouping that is based on students' academic abilities, was widely used in the United States school system from the 1960s to 1980s (Steenbergen-Hu, et al., 2016). From the mid-1980s to the 1990s, educators advocated for equity and equality which resulted in a decreased use of ability grouping (Steenbergen-Hu, et al., 2016). Like ability grouping, tracking is another way of grouping students based on their academics and, for the purpose of this study, will be identified as students following a specific academic route, resulting in a more permanent form of grouping as students are unable to change until the completion of a course. Researchers, such as Oakes (1985) and Slavin (1993), began to oppose ability grouping and tracking, claiming that tracking limited educational opportunities for some students. Despite the observed changes by the end of the 1990s, the United States began to see a rise in tracking, specifically for math (Steenbergen-Hu et al., 2016).

Stein et al. (1996) explained that in order for grouping to be effective, a teacher must consider more than simply seating children around a table. Important to note is that students are often placed in groups within the classroom, without consideration of any effective strategies (Kutnick & Blatchford, 2014). However, one aspect that does seem to have an influence on how groups are structured is culture. Interestingly, one study found that groups were often formed based on the expectations and norms of the country's education system (Sahlberg, 2011). Two examples exist in the countries of Finland and Japan where both are regarded as having different academic approaches, yet also have both been considered mathematically strong according to the Program for International Student Assessment (PISA). According to Sahlberg (2011), "Finland, a highly successful country in terms of international comparative testing, chooses to group students heterogeneously for the majority of their school" (p. 76). Similarly, ability grouping is commonly utilized in Singapore and New Zealand (Anthony & Hunter, 2017). Conversely, Japan determines their groupings by effort rather than by ability due to concerns regarding the negative impacts on a child's self-image (Sahlberg, 2011). Given that both approaches have seemingly proved successful in various areas around the world, this information only further supports that further research and exploration into the optimal approach is needed.

When it comes to the teaching of math, prevailing literature explores the development of groups (Tuckman & Jensen, 1977; Brenner, 2010; Wood & Kalinec, 2012) and compares group work between grades (Ramist et al., 1994; Mulkey et al., 2005; Mulryan, 2010; William & Bartholomew, 2013), but rarely examines the effects of structuring groups amongst students in one grade, or even in one specific subject. This warrants further investigation into the effects of group structuring in mathematical performance with elementary school students.

My desire was to first determine whether group work was beneficial for grade three students' mathematics, while also considering the way in which groups were structured. As previously identified, the group structures this study considered were peer-based groups and academic-based groups. While the use of academic-based groups is common in mathematics (Steenbergen-Hu et al., 2016), purposely placing students with their closest peers is not.

The reason for exploring the use of peer-based groups comes from my experience at teaching at an independent school. In British Columbia, students have the choice of attending two different types of schools: public or independent (Province of BC, 2021). Students who attend public school, are able to go free-of-charge, attending the school in closest proximity to their home (Province of BC, 2021). Alternatively, some families prefer to have their child attend an independent school, which they commute to; because of religious beliefs, teaching methods or programs offered (Province of BC, 2021). All schools K-12 are regulated by the B.C Ministry of Education and require teaching of the same BC Curriculum. School X, where the study took place, is an independent school that provides education for students from preschool to grade 12.

Teaching at an independent school, I have noticed that students exhibit a strong level of peer relationships. According to Hall (2019), in order for students to transition from being “acquaintances” into “friends”, they must spend at least 57 hours together. To transition from “friends” to “good friends”, an additional 119 hours of time spent together is needed (Hall, 2019). This would seemingly suggest that peer relationships develop from the increased amount of close contact. School X reported that 70% of the students who were in grade three during the time of the study, had attended since kindergarten. Conversely, this is not the case with students from public schools, as my conversations with colleagues who teach in public schools have confirmed; it is rare to have a majority of students return to the same school. It is important to acknowledge that this information has been gained through anecdotal conversations, as the Surrey School District was not able to provide any data on the number of students who attend the same school for all primary grades. Therefore, as School X provided a unique environment, in comparison to the neighboring public schools, I wanted to explore the use of peer-based groups in a situation where students have already had the opportunity build strong relationships.

Research Question & Aims

Based on the issues identified, the main research question for this study is listed as follows: How does the structuring of groups affect academic progress in math for grade three students in an independent school in Surrey, BC? This pilot study aimed to identify whether providing the opportunity for students to work collaboratively (through peer-based groups and academic-based groups), when compared to students who would work independently for all math blocks (control group), would have a positive impact on their academic achievements. Data will be gathered on existing research about the social learning, biological cognitive development, and the influence peers have on one another to gain insight into the importance of collaborative learning. Furthermore, research will outline additional moderators that could impact a student's ability to perform academically, while identifying the importance of group work in accordance with the BC Curriculum. In addition to the main research question, this study consists of the following supporting research questions:

1. Does the age of a student in the group impact his/her mathematical performance?
2. How does the composition of sex (i.e., same sex vs. mixed) in groups impact grade three students' mathematical performance?

Theoretical Framework

As this study considers differing groups and their influence on academic functioning in math, it promotes potential conclusions relevant to social learning and group interaction. Therefore, logically, this study is intended to encompass ideas proposed by social constructivism as its overarching meta-framework. Social constructivism is a philosophical stance that acknowledges the importance of providing opportunity for both social interaction, as well as, individual meaning to occur in order for a true understanding to be gained (Ernest, 1994).

Early theorists, such as Piaget (1965) and Vygotsky (1978), have recognized the importance of developing mathematical skills and the influence that peers have in early ages. Vygotsky's work can be grouped into three primary categories: the best way to understand the mind is to look at how it changes; higher mental functions originate in social activity; and higher mental functions are developed by using tools and signs (Vygotsky 1978, as citied in Hausfather, 1996). Specifically, Vygotsky (1978) proposed that children's cognitive development takes place largely because they interact with their environment and with others. Furthermore, Piaget (1965) recognized the influence that friends have over learning. If one considers that a student typically spends a minimum of 13 years in school, there are a myriad of learning opportunities that arise when interacting with peers (e.g., Willms, 1986). Piaget acknowledges that when peers are provided with the opportunity to work together, it promotes learning (Piaget, 1965, as citied in DeLay et al., 2016). Therefore, it appears evident that the noting of social learning, which includes the concept of working collaboratively with peers is not new to our education systems and has also been viewed as beneficial for many years. As a result, developing an understanding of the way groups are structured could be further benefit a student's learning experience.

Vygotsky's (1978) view on social constructivism acknowledges that when students are given the opportunity to work with others, what they are able to produce is more indicative of their mental development than what they do on their own. This notion of the importance of social interaction in learning encounters, underscores Vygotsky's (1978) concept of the Zone of Proximal Development (ZPD), which is defined as "the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers" (p. 86). In order to further development via social interaction, one strategy that has been commonly used is scaffolding. Scaffolding is the process of designing incremental steps in the learning process where the students are provided with guidance and examples until they are able to complete a task on their own and proceed to the next step (Sanders, 2005). Following Vygotsky's initial ideas regarding the ZPD, Bruner (2020) also identified the importance of working with peers as he found that students would learn better when given the opportunity to discover and learn concepts on their own, as opposed to, being given the information through lecture style learning (Marsh & Ketterer, 2005).

In addition to seminal works from Vygotsky and Piaget, the theoretical framework on which this study is based also comes from recent researchers such Jo Boaler and Carol Dweck.

Boaler, who is currently a Professor of Education at Standford University – Palo Alto, has devoted her research to helping students and educators experience the importance of having a "growth mindset" in mathematics (Boaler, 2016). Boaler's understanding of a growth mindset in mathematics came from research conducted by Carol Dweck, who is a psychologist at Standford University (Heggart, 2015). According to Dweck, when students have a fixed mindset, they believe that their basic abilities, their intelligence, their talents are just fixed traits – they only have a certain amount of intelligences or a certain set of talents (Dweck, 2017). However, when students approach learning with a growth mindset, they understand that their talents and abilities can develop through effort, good teaching, and persistence work (Dweck, 2017).

Boaler (2016) specifically used the concept of having a growth mindset and related it to mathematics. Math, more so than any other subject, has a culture of being viewed as a subject that only some people can excel in. In fact, there is a common misconception that some students are not developmentally ready for certain types of mathematics (Boaler, 2016), although they inherently would need to be age appropriate. From my observations in the classroom, students are often fearful of making a mistake in math, however, it has been found that simply performing mathematical tasks can benefit students academically. When students make a mistake in math, even if they do not get the answer correct later, a synapse grows (Dweck, 2006). This was further explained by Moser et al. (2011) as he found that when mistakes occur, it is during this time that the brain grows the most as it is intellectually challenged. One of the reasons for this is due to having a growth mindset because students no longer become fearful of making mistakes; and as they work through mathematical problems, students develop a greater awareness for when mistakes are made and are more likely to go back and correct their errors (Magnels et al., 2006). Therefore, this shows that for learning to occur, students do not need to always get the correct answer, but can also learn through the process of solving a problem and increasing their awareness of when corrections need to be made.

According to one of the top mathematicians, Kevin Devlin, math at its core is the study of patterns – suggesting it is rather an aesthetic and creative subject (Devlin, 1997). Often our perspective of math involves numbers and formulas; however, at its core, the understanding of numbers involves discovering patterns and ways to manipulate numbers which requires more than the ability to regurgitate a concept taught on a whiteboard. However, our current mathematic systems do not allow for much creativity and aesthetic appreciation. That is, instead of mathematics being a subject of exploration, it has become one in which memorization is required and praise occurs when students are able to come up with the correct answer as fast as possible (Boaler, 2016).

When social learning is incorporated into classrooms and students are given the opportunity to work collaboratively, math then becomes a subject for exploration as students show an increased willingness to discover multiple ways of solving problems (Boaler, 2016). I have observed this especially when students are given the opportunity to solve word problems together, as students are able to show their learning in various ways instead of just being required to fill in a box with the correct answer. When a shift between a fixed mindset, where math is simply about following a formula to get the right answer to a growth mindset occurs, it allows for curiosity, connection making, challenges and creativity, and students naturally begin working collaboratively (Boaler, 2016).

When students show a willingness to change their mindset from a fixed mindset to a growth mindset, their understanding of numbers, referred to as number sense, can also develop (Dehaene, 2011). Number sense looks at how the human brain interprets, which in turn helps us make sense of numbers and mathematics (Dehaene, 2011). For example, when students are younger, they are able to understand that one of their siblings got more pizza than the other, however as they get older, they would be able to represent this concept through a fraction.

Figure A1 provide examples of what can happen when students are given the opportunity to work collaboratively and how students understanding of numbers can greatly vary, while all showing the correct answer.

Figure A1

Visual Solution to 18 x 5[1]

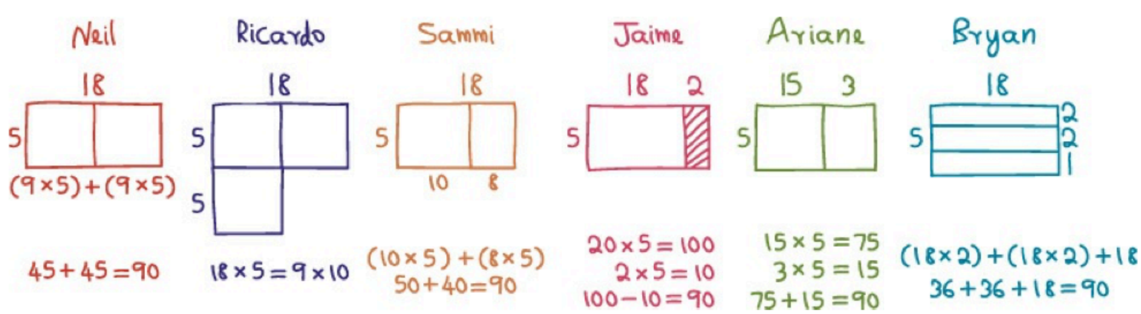


Figure A1 is the result of a question proposed during a visit between Jo Boaler and Sebastian Thrun in December 2012. At the time, Thrun, was in the midst of creating online courses and questioned Boaler as to why students were failing math. Boaler acknowledge that the reason students were failing was not due to the difficulty of the mathematical concepts, but instead due to a lack of number sense (Boaler, 2016). Boaler asked students to work collaboratively to show (Figure A1) how 18 x 5 could be represented. Instead of simply regurgitating a formula or pattern taught by their teacher, students worked to come up with their ideas of how to solve the program. This is the reason that mathematics is being used as an avenue for exploring social

[1] Boaler, 2016, pg. 59

learning in this study. By incorporating the use of social learning, the desired outcome is to see how academics are influenced when students are given the opportunity to work collaboratively in mathematics.

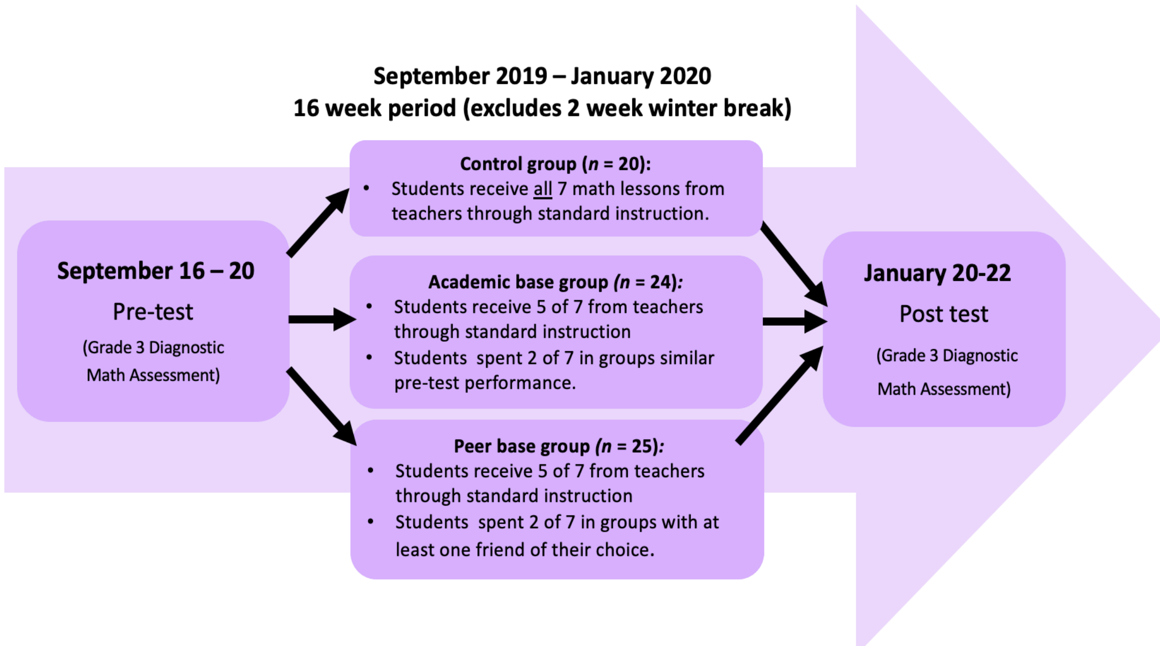
Overview of Research Methodology

This study explores the benefits of a collaborative approach to mathematics. Therefore, the desired outcome is to determine if students' academic mathematical ability increases when students are given the opportunity to work collaboratively, followed by investigation into whether the group structure (peer-based vs academic-based) affects mathematical performance over time. The utilized research design and methodology are outlined below.

Research Design. The present study is loosely based on a quasi-experiment intervention study, where groups were structured following the pre-test diagnostic assessment. Hence, this is a three (Group: Peer-based, Academic-based, Control) by two (Test: Pre-test, post-test) between-within subjects design. The two intervention types were academic-based groups and peer-based groups. Groups were purposefully structured to determine the benefits of students being placed in academic-based groups versus peer-based groups. A detailed explanation on how groups were formed can be found in Chapter 3 (see pg. 67). Students first completed a pre-test measure, followed by 16 weeks of JUMP Math (see below for an explanation on JUMP Math), concluding with a post-test (see Figure A2). The Vancouver Island Numeracy Test was used for the pre-test and post-test. Further discussion on the instrument tool, reliability, and validity, as well as, an adaptation can be found in Chapter 3 (see pg. 60).

Figure A2

Experimental Procedure



All students received access to the same content and were provided with the same amount of math blocks[2]. However, the format of lessons varied between the intervention groups and control group to provide the opportunity for social learning in the form of collaborative work.

Junior Undiscovered Math Prodigies (JUMP Math), which is a program developed by John Mighton, was already implemented and used by School X as a resource for teaching math concepts (Jump Math, 2002). The JUMP Math program is a K-8 Canadian mathematics program (Jump Math, 2002)[3]. Each grade level has two workbooks (i.e. 3.1 & 3.2) where pictures and mental strategies are used as concepts of scaffolding until students gain mastery of a concept (Jump Math, 2002). Using the JUMP Math program was not imperative for this study, as there

[2] Each math lesson approximated to 40 minutes of instruction and work time.

[3] Further information on the JUMP Math program can be found at https://jumpmath.org/ca/.

was no consideration regarding the quality of math program that was used. This study looked strictly at the impact of structuring groups and mathematical performance. However, as JUMP Math was already used at School X, the use of this program was not only necessary, but also addressed potential confounding variables such as teaching methods and practice materials. In order to keep consistency amongst the three different classes, each lesson began with teacher instruction, which was guided by the teacher manual that coincides with the JUMP Math curriculum. While learning the concepts and content of JUMP Math, all participants used the same workbook for practice. However, compared to the control group, academic-based and peer-based groups were given two blocks a week to work collaboratively within their specified groups.

Sample

The sample consisted of grade three students attending School X in Surrey, BC, which included three subgroups determined by the previous year's teachers. In order to determine the demographics of the sample, parents were asked to fill out a questionnaire as a way of self-reporting. Through these results (see Appendix H), this sample is considered fairly homogenous in two respects: socio economic status (SES), parents' education (Appendix H8), marital status of parents (Appendix H9); and learning challenges as represented through an Individualized Education Plan or Learning Plan (Appendix H10). Parents were not required to share SES as School X is a private school where tuition is required; therefore, it can be argued that families come from a similar enough SES background for the purpose of this study. In comparison, the sample is heterogenous when ethnicity (Appendix H4), language (Appendix H5), and family composition (Appendix H6 & H7) are considered.

As I currently teach grade three, convenience sampling (i.e., accessing a sample of individuals who are readily available to participate in studies) was used (Etikan, Musa & Alkassim, 2016). While convenience sampling has allowed me to gain further insight in this educational setting (e.g., improving personal teaching practices, sharing first-hand knowledge with colleagues), this design could raise concerns for sampling bias (Taherdoost, 2016). Sampling bias is a systematic error if pre-existing differences in the sample interact with variables (i.e., internal validity), or if the participants that are chosen favour certain characteristics of a group (i.e., external validity) (Spieth et al., 2016). I contained sampling bias in that I assured the internal validity of the study using triangulation for results by accounting for demographics (i.e., ethnicity, gender).

Data Collection Methods

The Vancouver Island Numeracy Assessment (INA), which was the tool used as a pre- and post-test, was given to each subgroup in their individual classes (Island Numeracy, 2020). The measurement is discussed in more detail in chapter three (see p.54). The INA was used out of convenience as it was commonly used at the beginning of the school year for School X, covers relevant concepts for grade three curriculum and is easily accessible for all teachers. Each classroom teacher was given a script to read, which ensured consistency with instruction, time limits and the teacher's ability to help students.

Data Processing and Analysis

Once the test was administrated, it was scored and the raw scores were recorded on an Microsoft Excel spreadsheet. All tests were scored by me using the answer key, which is provided. These results were then put into the Statistical Package for Social Sciences (SPSS) to run the statistical analysis.

To gain a better understanding of the participants, a questionnaire was filled out by parents. To determine if there was a difference between groups, a repeated mixed design ANOVA was used. In doing this, an understanding of the repeated measure (pre- and post-test) was considered, as well as, two between-subject factors: gender and age. Time, sex and age were compared across the three groups (control, academic-based and peer-based).

Listed below are the following hypotheses followed by a rationale:

Hypothesis One. An improvement will be observed between the pre-test score and post-test scores as a result of time.

Rationale. When students are exposed to ongoing mathematical content, an improvement is observed in their abilities (Rotherham & Willingham, 2009). As the pre-test was done in September, where no exposure had been given to grade three mathematics, it was expected that an increase in ability would be observed 16 weeks later.

Hypothesis Two. As measured by post-test INA mean scores, students in the peer-based group are expected to score higher than students in the academic-based group and the control group. Meanwhile, students in the academic-based group are expected to score below the peer-based group, yet higher than the control group.

Rationale. Many countries (e.g., Japan, Finland, etc.) that have been known to be highly successful in mathematics have taken the approach of grouping students as part of their mathematical practice (Sahlberg, 2011). Providing a specific time for students to work together allows them to collaborate and learn from each other.

Hypothesis Three. Students in the peer-based group will show a greater pre-test to post-test mean score improvement (as measured by an increase in the INA) than those who are placed in the academic-based group as a result of being able to work with peers who they are already comfortable with.

Rationale. A student's perception of themselves has been known to have a direct effect on their academic competence (Eccles et al., 1984). Therefore, when students are placed with their peers, there is a great chance that they will feel supported, producing a greater influence upon their academic abilities.

Ethics

I received approval from the Research Ethics Committee at Trinity Western University (Appendix F). Prior to study, an introduction letter was given to the parents or legal guardians of all potential participants. As all participants were minors, parental consent was required (Appendix E). Each teacher, through a script, relayed to all students that the participation in the study was not mandatory and that they had the right to withdraw, which is a norm when researching children (Fleet & Harcourt, 2018). By providing each student and parent with the opportunity to withdraw, non-maleficence was considered.

All assessment results, parental consent and parental questionnaires were kept in a locked storage area, where I was the only one with a key. Each participant was assigned an identification number, which was used when recording all data, providing anonymity of all subjects. Data was stored on a computer and could only be accessed by inputting a password.

Upon completion of the study, parents will be provided with the opportunity to read this document. Teachers at School X, as a way of professional development, will be invited to listen to the results. By providing these events, both teachers and parents may gain knowledge into how the structuring of groups affects grade three students.

Demarcation of this Study

The study consists of the following chapters:

Chapter One serves as an introduction to the study and outlines the intended outcomes by giving a brief overview of the study and acknowledging previous research conducted.

Chapter Two contains a detailed literature review by presenting recent theories, as well as, developing an understanding of how various countries have developed group structuring to provide an opportunity for all students.

Chapter Three presents the research design and methodology of this study. It also acknowledges how the data was collected and assessed.

Chapter Four presents the results of the assessment and analysis of the results.

Chapter Five concludes the study with a discussion of the results, a summary, and acknowledgment of the limitations and further recommendations.

Conclusion

This chapter described the background and rationale for this study. The next chapter will present a detailed discussion of previous literature by reviewing and synthesizing theories and past examples of those examining the use of collaboration within mathematics.

Chapter 2: Literature Review

What follows below is a vignette based on personal experience that informed the research interested in exploring the way in which groups are structured. Suzy and Molly are in the same class. Suzy is exceptional in math, but Molly has been struggling to learn the concepts. The teacher paired Suzy with another exceptional math student named Jacob. This leaves Molly with Jenny, who is also struggling in math. As the teacher is unable to work with Molly and Jenny, she decreases the amount and difficulty of their work and places students in pairs in hopes they will be able to finish their work in the given time. However, this means that Molly and Jenny are no longer exposed to the more difficult questions and will not receive the same amount of practice.

This vignette highlights an important aspect of teaching that I have witnessed in my own classes, and amongst my peers who teach kindergarten to grade seven. We group students to support their learning. However, it raises the questions of how well we, as educators, are meeting the mathematical needs of all students and whether there might be a better method of how we structure group work.

Social learning has recently become a focus in BC through the use of core competencies, which include communication, thinking, and personal and social skills (BC Curriculum, 2020). According to BC's Curriculum, core competencies develop intellectual, personal, social, and emotional skills with the desired outcome of creating engaged lifelong learners (BC Curriculum, 2020). One way in which social learning is achieved is through group work (Boaler, 2016).

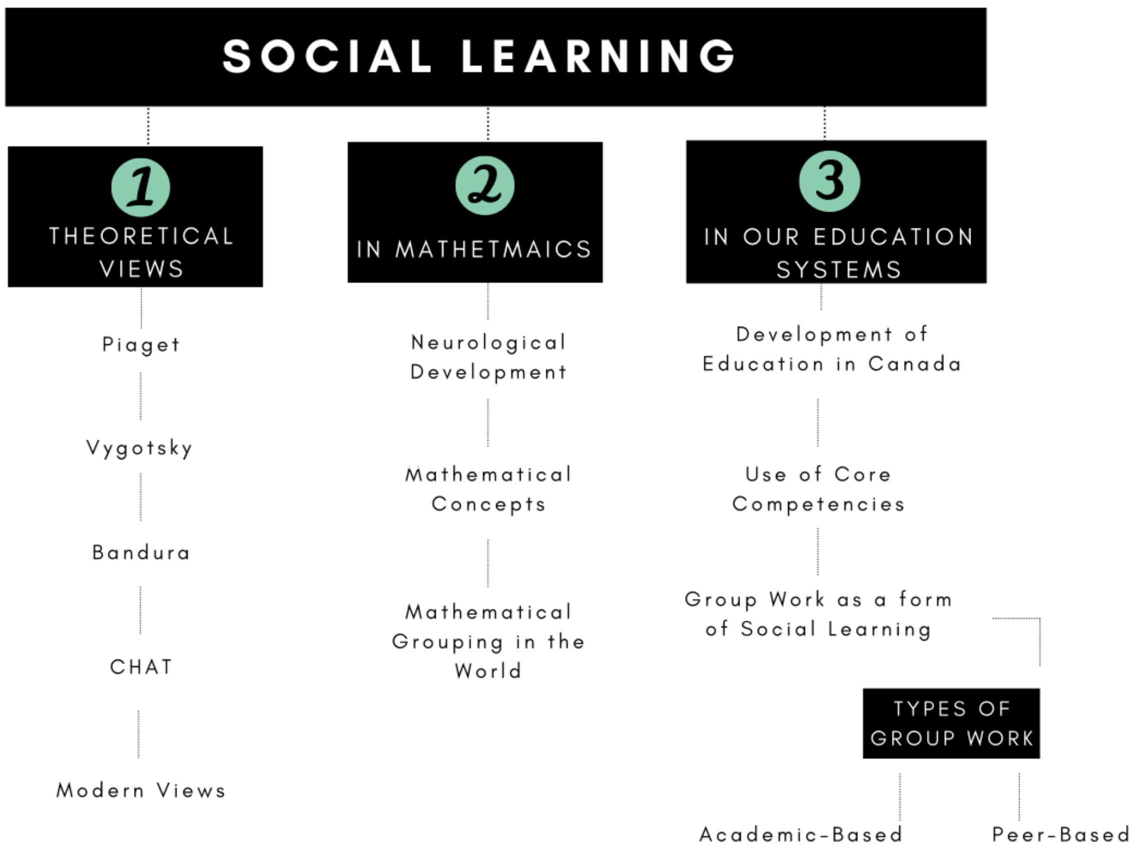
To gain an understanding of the importance of social learning, this chapter will outline three main areas: the theoretical views of social learning, an understanding of social learning in mathematics, and an insight into how social learning is currently being used in our education systems.

Math is an invaluable skill that can enhance the way in which individuals' function both academically and in work fields (Wolley-Wilson, 2013). As math is predominantly developed in primary school education, it has been recognized that the earlier a child can acquire math skills, the more likely he or she will be to achieve success in math (Christensen, 2011). It has been identified that for children to be successful in mathematics, they require lessons that provide understanding of the content, repeated exposure, and the opportunity to practice their skills (Cox, 2004; Mason, 2001; Leonard et al., 2010). However, the way in which this is achieved can differ. Therefore, this study will use mathematics as an avenue to explore the importance of social learning in the classroom while also looking at the use of two different ways in which group work can be achieved.

Currently, education systems around the world have been observed to group students through the use of tracking (New Zealand; Anthonly & Hunter, 2017) and ability grouping (Signapore; Kaur, 2014), while also placing students in heterogenous groupings (Finland; Sahlberg, 2011). In order to further an understanding of social learning, through the lenses of group work, this chapter will also explore: the use of mathematical grouping in the world and the advantages and disadvantages of academic-based groups and peer-based groups. Figure B1 has been developed as a visual representation to guide you through Chapter 2.

Figure B1

Visual Representation of Chapter 2 Topics



As depicted in Figure B1, this chapter is divided into three main topics (as observed through the number boxes) with various subtopics.

Theoretical Views

Social constructivism examines how knowledge and understanding of the world are developed jointly by individuals, as knowledge is not only gained independently, but also through experiences and opportunity with others (Fox, 2001). Seminal works by Vygotsky and Piaget prompted and encouraged the progression of social learning. Vygotsky (1978) explained that cognitive growth needs to first develop on a social level followed by individual knowledge as students must be able to relate circumstances to themselves (Roth, 2000), whereas Piaget's focus on the concrete operational period, which occurs from age seven to eleven, emphasized the importance of developing mathematical skills (Vergnaud, 2009). According to Vergnaud (2009), the reason for this is that according to Piaget, it is at this stage that students are able to develop logic and reasoning skills, allowing them to understand process required for mathematics. This pilot study involves participants who were predominately seven and eight years old. Considering Piaget's concrete operational stage is important because it shows that students are at an age in which mathematical skills can be developed and therefore looking at the best method for skills to be taught is beneficial to our current education systems. Below an explanation of seminal works, as well as, modern day views will be provided as way of understanding how social learning has come to be.

Piaget. Jean Piaget explained the way in which children develop via a series of four stages: The sensorimotor stage (birth - 2 years), pre-operational stage (2 - 7 years), concrete operational stage (7 - 11 years) and formal operational stage (12 - 16 years) (Piaget, 1973). Piaget believed that children should not be taught certain concepts until they reached an adequate stage of cognitive development (Piaget, 1964). In the concrete operational stage, children develop skills in logic, reasoning, resolving problems, and the ability to think through difficulties in a logical way (Piaget, 1964) The concrete operational stage is most relevant for this study as it examines the development that occurs between the ages of seven and eleven, corresponding to our participants being in grade three. In conjunction with this, the concrete level also includes development in the area of classification, ordering, construction ideas, spatial operations and relationships in mathematics (Ghazi, 2015).

Many children require lessons that allow for conceptual meaning-making, as opposed to just being given the concepts (Vergnaud, 2009). However, teachers sometimes still resort to a method of rote-teaching. Rote-teaching is observed when the teacher stands at the front of the class giving examples, resulting in limited interaction and minimal opportunities to make mental connections with concepts (Vergnaud, 2009). Piaget's identification of the concrete operational period emphasizes the importance of developing mathematical skills through activities involving objects (Piaget, 1968). Although active construction of learning has been widely adopted in England, Australia, and the United States in primary and middle school, the traditional learning method of the teacher as the "sage on the stage" is still common within our education systems (Vergnaud, 2009).

Vygotsky. Similar to Piaget's view on active construction of concepts, Vygotsky also recognized the importance of meeting the learning needs of children, based not only on their developmental level, but also on their interactions with a more knowledgeable peer or an adult (Vygotksy, 1978). In fact, scaffolding (Sanders, 2005) and mentoring (Kozulin et al., 2003) are important supports for student learning, based on Vygotsky's Zone of Proximal Development (ZPD). Vygotsky (1978) defined ZPD as "the distance between the actual development level as determined by independent problem-solving and the level of potential development as determined through the problem-solving under adult guidance or in collaboration with more capable peers" (p. 86). Vygotsky's notion of ZPD has been largely influential in the way in which we can perceive collaboration, as he claimed that individuals learn best when they are provided with the opportunity to work with others, especially with those who are more skilled, as it can lead to learning new concepts and skills (Vygtosky, 1978).

Vygotsky also believed that children needed to be guided and supported, which can be obtained from the instruction, including the *More Knowledgeable Other* (MKO) (Vygotsky, 1978). According to Vygotsky, a MKO is a person who has a greater knowledge and skills than the person learning and can provide social instruction with the learner (Vygtosky, 1978). Therefore, when children are provided with mentorship like the MKO (e.g. a teacher, tutor, or more advanced peer), they are being met and assisted at an appropriate level for their development and learning (Kozulin et al., 2003).

Cultural-historical activity theory (CHAT). Social constructivism, a term introduced by Vygotsky, acknowledges that in order for learning to occur, students need to be provided with social interactions, leading to learning in a collaborative manner (Fox, 2001). This can occur, but is not limited to, partner work, problem solving and large group work. Like social constructivism, Cultural-Historical Activity Theory (CHAT) is a s theory is based on the concept that the human mind is not located within the brain, but instead "the mind is in actions and activities in which humans engage with the world" (Sanino & Engestrom, 2018, p. 44). CHAT provides a framework that allows for this to be done as it involves three primary ideas (Engestrom, 1987):

1. Humans act collectively and learn by doing and communicating through their actions
2. Humans will use and adapt tools to learn and communicate
3. Community is required to process and learn

CHAT shifts our perceptions of what happens inside the individual to what happens between humans in a cooperative manner (Sannino & Engestrom, 2018). Our current education systems have placed an emphasis on the use of collaborative work as educators have now been asked to diversify instruction resulting in a decrease of direct instruction and an increase in collaborative work (Talanquer, Tomanek & Novodvorsky, 2013). In this study, the focus is on the object of the activity (in this case, the learning of math), the organization of the activity (in this case, group work) and on whether an increase in academic achievement is observed when provided with the opportunity for collaborative group work.

Bandura. Given that children are social by nature, it may explain why social interactions are vital to their development at each stage in their life (Bandura, 1977). My observations as a teacher have revealed that learning encompasses social interaction and the classroom involves cognitive and behavioural processes (e.g., listening to discussions, talking to others, applying insight to other students 'ideas, etc.). These observations fit with Albert Bandura's Social Learning Theory, which identified that by observing other people, people are likely to imitate and develop similar behaviours (Bandura, 1977).

Social learning theory may offer interesting perspectives on the current classroom learning and the teaching environment. In everyday situations, social learning provides a greater understanding of how ideas raised in educational settings quickly spread across a group (Hill et al., 2009). However, in mathematics, it was found that teachers often complete a few problems with limited explanation or interaction with students (Fuson, 2009). This traditional learning style, referred to as rote-learning methods, fails to reach the needs of many students (Fuson, 2009). The reason for this that often textbooks approach mathematical concepts with an isolated, simplistic view, which has resulted in two major problems (Boaler, 2016). Firstly, students are practicing concepts in a simplified form, which does not provide the opportunity to make connections or have a sense of meaning. Secondly, from my observations, when students are required to practice the same concept over and over, boredom increases and the desire to finish as quick as possible increases. This is one reason why students struggle with word problems.

Although they can complete questions in their workbooks, showing the ability to answer a specific question in a specific way, students have not developed an understanding of how to use numbers conceptually, in a flexible format (Boaler, 2016).

Modern Views. The benefits of social learning have been observed for many years, starting with seminal works from Vygotsky and Piaget, which have continued to influence our modern-day systems of education. In the 1930s, Piaget acknowledges that learning is not about memorizing, but instead about understanding how ideas fit together (Piaget, 1930). Unfortunately, our current education system has moved away from this notion in mathematics, as often the focus instead is on simply completing steps and calculations in a quick manner (Boaler, 2016). Conrad Wolfram, the director of Wolfram-Alpha, has criticized this concept and proposed that working on mathematics should rather consist of four stages (Wolfram, 2010):

1. Posing a question
2. Going from real-world to a mathematical model
3. Performing a calculation
4. Going from the model back to the real world, to see if the original question was answered

Students currently describe math as a subject of calculations, procedures, and rules. Eighty percent of students in the United States claim that their math classes are focused on Wolfram's third step and rarely incorporate questions, however, the ability to ask questions and gain understanding through discussion is vital to all subjects as students rarely gain an understanding of a concept without talking it through (Boaler, 2016).

John Hattie (e.g. Hattie et al., 2017), who is known for his research on visible learning, has used effect size to determine which methods have the greatest influence on learning.

According to Hattie et al. (2017), whose research is based on data from over 800 meta-analyses around the world, when the effect size is greater than 0.40, the method of learning can be considered to have a desired effect. Classroom discussions, which are rarely observed in classrooms (Wolfram, 2010), were found to have an effect size of 0.82, as they provide opportunities for students to help one another and make connections in mathematics. Furthermore, peer tutoring, which often occurs when students are provided with the opportunity for social learning, was also observed to have a strong influence with an effect size of 0.55.

Social learning may not only benefits students' learning, but also provides a way to incorporate the core competency of *communication* that many Canadian provinces have deemed an essential part of education. The reason for this is that one form of social learning is collaboration. Collaborative work encourages communication amongst group members and foster learning as students discuss and collaborate (Hill et al., 2009).

Some concluding remarks: Research and knowledge gained from the discussed seminal works and theorists have shaped, and will continue to guide, the way in which we approach education. When considering work conducted by seminal authors (ie.Vygotsky), it is important to recognize that our education system is reflective of our societal needs, and therefore, different than the needs in the early 20th century. However, despite time frames and differences in culture and societal influences, I would argue that looking at the methods and theories of learning in the past is vital to understanding our current systems. The method in which information is obtained and used has greatly changed (ie. textbooks vs computers), however, not all aspects of learning have. One aspect that has not changed is that school requires social interaction, and therefore, social learning encounters and no one single author is more of an authority on this than Vygotsky. As students are placed in classrooms together and are learning in social interaction, investigating former theories, as well as, current research, provides a greater understanding of the challenges and successes involved in learning in this way.

As this study is using mathematics as an avenue for discussing social learning, the next section will discuss the neurological developments required for mathematics, the mathematical concepts discussed in grade three and provide an understanding of how mathematics is currently being conducted in the world. This will provide a framework for understanding how social learning can contribute to mathematics.

Social Learning in Mathematics

To understand how social learning occurs, this section will first explore concepts that influence mathematical development that teachers have limited control of (i.e. neurological development, age, and sex), followed by those which teachers influence (i.e. skills and concepts).

Neurological Development. The importance of adapting instruction to meet the needs of different age groups has been recognized as early as the twentieth century, when age-graded schools became the norm (Miller, 1991). To understand the influence of age, it is important to understand how the brain develops and matures, as it can influence cognitive behaviour (Nordqvist, 2017). Even though neurological development is not the main focus of this study, this section will briefly outline childhood development as it provides a broader perspective on how students learn mathematics.

According to Dehaene and colleagues (2006), the area of the brain that is associated with processing math information is the horizontal intra parietal sulcus (hIPs). Although the hIPs is the primary area associated with math, the pre-frontal cortex (PFC) also assists with additional understanding (e.g., shapes) (Blair & Raver, 2015). A child's brain undergoes significant development in early childhood (Blakemore, 2012; Giedd & Rapoport, 2010). During this time of synaptogenesis (i.e., the process of neural network strengthening) (Blakemore, 2012; Giedd & Rapoport, 2010), the brain produces more synapses than required; therefore, the synapses that are used less frequently become weaker than the ones regularly used (Giedd & Rapoport, 2010). As previously mentioned in Chapter 1 (see pg. 11), when children complete math questions, even if they do not reach the correct answer, a synapse develops. If a student becomes discouraged or claims to not like a subject, then it would appear the synapses involved in the concept becomes weaker. For example, if a student had become discouraged about their abilities to perform multiplication questions, they would likely be hesitant to try more advanced questions – resulting in a weaking of a synapse. This relates to my study as I have observed students who are weaker at a concept (ie. multiplication, fractions, decimals) often display an increased willingness to attempt questions when they are able to work with a friend. Even if they are working with a peer, the constant, repetitive exposure can assist in strengthening synapses (Giedd & Rapoport, 2010).

Chronological Age. When grade levels were established in the twentieth century, this approach assumed that students in each grade level had the same needs because they were in the same age bracket (Rodrigeuz, 2016). However, an arbitrary cut-off date may not accurately account for the different stages of development that can occur over a 12-month period.

In most countries, the educational administration defines a cut-off dates for school enrolment, which stipulates that a child has reached an age where they are considered ready for school. However, this results in an age range of up to 12 months within one grade. These differences are often evident when maturity levels or academic performance are considered (Rodrigeuz, 2016; Thoren et al., 2016).

When mathematics performance was considered, Yesil and colleagues (2012) found that children who had been delayed in regard to enrolment "had stronger mathematics skills than did on-time enrolled children, who had stronger skills than did the early enrolled children" (p. 3071). In BC, children are able to enter kindergarten as long as they turn five by December 31 of their enrollment year (Province of BC, 2021). However, parents can delay enrollment by one year, which is sometimes observed for children who have birthdates later in the year (ie. December) (Province of BC, 2021). Therefore, the term delayed enrollment refers to these students – those whom could have entered kindergarten the previous year, but as a result of parental decisions, were held back.

Similar results were discovered by Crawford et al. (2014), who gave the same assessment to grade one students once they researched a certain age in months and compared results to those who were tested at the same time of the year. It was shown that the younger students performed worse, therefore, suggesting the potential impact of age on academic performance. However, Thoren and colleagues (2016) found that a student is relative to the age effect. For example, when both mathematics and reading were considered from grade two to grade three, a decrease in effect in regard to age was already observed. By grade eight, the age effect had vanished, thus showing that age may have a greater impact on younger students (Thoren et al. 2016). Therefore, this shows that the impact of age is likely to have a greater infuence on students in primary grades (kindergarten – grade three). Therefore, chronological age was considered as a moderator for this study as all participants were in grade three.

Sex. In addition to age, sex is another factor that cannot be developed. Nosek, Banaji, and Greenwald (2002) acknowledged that mathematics is often stereotyped to be a male domain. This may be due to the negative emotions that are often observed from elementary teachers as this has been a predictor for girls 'achievement; however, this was not observed for boys (Beilock, et al., 2009). Mathematics, as opposed to other subjects, often puts an emphasis on speed – the notion that you must be *fast* to be *good* (i.e. timed math tests, flash cards, etc.) (Boaler, 2016). For many girls, the added pressure of performing quickly results in increased anxiety leading to worse performances, despite knowing the concepts; however, the severity of pressure and anxiety was not as evident in boys (Boaler, 2016).

Signorella, Frieze, and Hershey (1996) completed a 10-year longitudinal study of single- and mixed-sex classes in a private school, where they concluded that when single-sex classes were considered there were no consistent tendencies for gender stereotyping. Ellis (1894) who originally studied gender in private schools hypothesized that when comparing males and females, the variability of intellectual abilities would be greater in females and which was explained by phenomenon where "there was both an excess of males among the mentally defective and very few female geniuses" (Hyde, Mertz & Schekman, 2009, p. 8802). This phenomenon was a catalyst for Li and colleagues' (2018) research which analyzed gender difference amongst grades five to eight in Beijing ($N = 73{,}318$). No sex differences were observed for grade five students, with a relatively small sex difference for grade eight students, resulting in females scoring slightly higher (Li, et al., 2018). Results revealed a great variance in scores for male students. Therefore, it appears that sex may not have a large influence in early elementary; however, it is something that should be considered in older grades. By coincidence, one difference between peer-based groups and the academic-base group in this study was that they were divided by sex as the peer-based group was divided between sexes and the academic-based group consisted of mixed sexes. This is the reason that the difference between males and females was explored as a moderator.

Influence of the Teacher on math learning

Biological and neurological differences can have an influence on students; however, students are also influenced by contextual factors. These include home environment, school environment, teacher-variables, and peer interaction (Hattie, 2003). Throughout a North American student's education, one will spend around 15,000 hours at school and interact with approximately 50 different teachers (Hattie, 2018). Teachers have a large influence on students – not only on their academic abilities, but also on their mindset (i.e. self-perception, confidence, etc.) (Cohen & Gracia, 2014; Boaler, 2016; Hattie, 2018). Using Hattie's effect size (d=0.72), the importance of the teacher-student relationship has again been recognized as influential to a student's success (Hattie, 2017). When students have a growth mindset, their ability to learn increases as they are not afraid to make mistakes (Boaler, 2016). Although teachers do not have an influence on biological factors, it is evident that they have a strong impact on a student's mindset, which can greatly influence their ability to learn. The next section will provide an understanding of the areas and concepts that teachers do have the ability to influence.

Executive Functioning. One area of note in childhood cognitive developmental outcomes, is executive functioning. Blair and Raver (2015) acknowledge that a child's pre-frontal cortex significantly matures around the age of five or six; therefore, children increase the use of their executive functioning (EF) skills during this time. EF skills consist of three different cognitive outcomes: working memory, impulse control, and cognitive flexibility (Miyake et al., 2000; Blair & Raver, 2015).

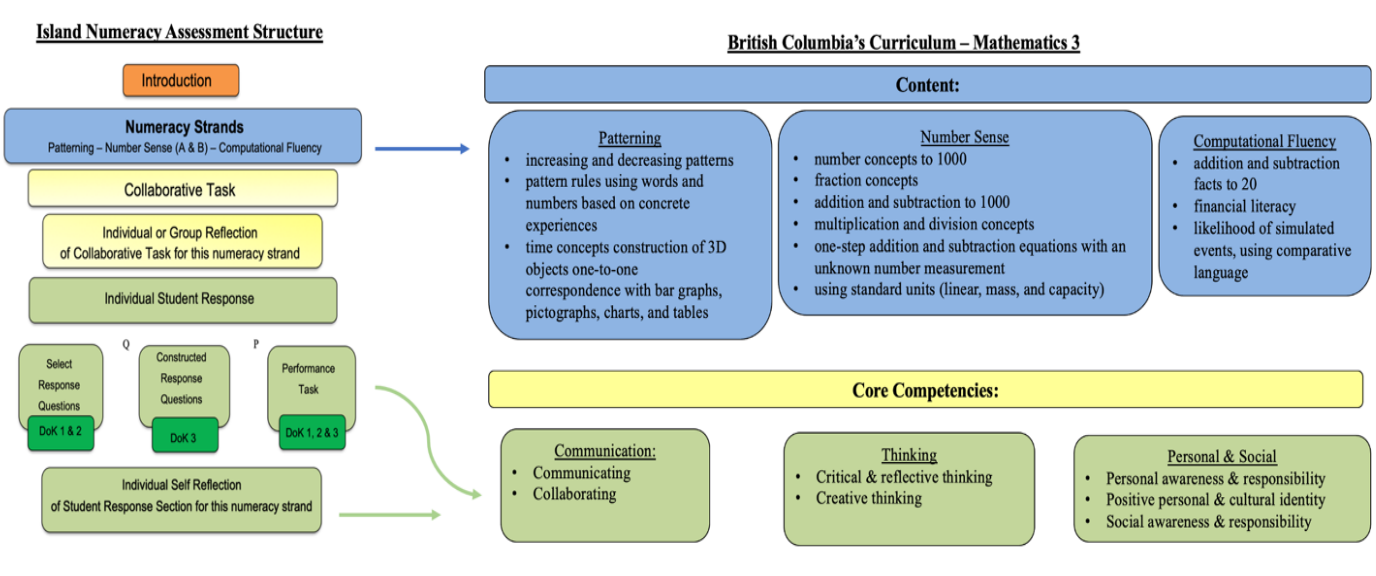
The connection between EF skills and mathematics is well established amongst researchers (e.g. Bull, 2014; Blair & Raver, 2015). One explanation for this holds that math requires the use of cognitive processes, which is important as students learn new concepts and skills (Watson & Gable, 2010). Another explanation is that EF skills help students develop the ability to shift between concepts, otherwise known as cognitive flexibility (Bull, 2014). This has been shown to lead to academic success, especially during academic tasks that require multiple steps, with concepts that involve planning, and with activities where guiding is a component (Best et al., 2009). In addition to academic success, EFs have been linked with classroom engagement (Fitzpatrick & Pagani, 2013; Robinson & Mueller, 2014). Therefore, the following suggests that strong EF skills can assist a student academically. However, they may not only assist with individual performance, but can also improve students' abilities to interact with peers. The reason being that EF skills can assist with developing the cognitive processes needed for impulse control and flexibility, which are both concepts needed when working with others (Blair & Raver, 2015), in for instance, following rules in activities such as turn-taking.

Although a connection between EF skills and mathematics has been recognized, an understanding of specific mathematical concepts may provide a greater understanding of how social learning can enhance a student's learning experiences.

Mathematical Concepts. In grade three, students are required to learn specific mathematical concepts which build upon knowledge from previous years. The BC Curriculum outlines three main areas students need to develop: Core Competencies, Big Ideas, and Content; otherwise referred to as "link[ing] the knowing, doing, and understanding" (BC Curriculum, 2020). The Curricular Competencies are meant to help students develop the skills to know what to do with their content knowledge in mathematics (BC Curriculum, 2020).

Figure B2

My Comparison of BC Mathematic Curriculum and INA



Note. The INA covers the content and core competences for BC Curriculum as identified through colour coding.

Whereas big ideas and content address academic-related skills and concepts that have been assigned to specific grades (BC Curriculum, 2020). I have summarized the relationship (see Figure B2) between the Core Competencies, Big Ideas, Curricular Competency and Content, for grade three students in conjunction with the objects of the Vancouver Island Numeracy Assessment (INA), which was used for my pre- and post-test.

This study used the 2007 INA, for an assessment tool, as the 2007 INA was the only version available at the time; however, the 2020 version was developed shortly after the study commenced. Table B1 shows the similarities in content between the two assessment tools, which demonstrates that number sense and computation fluency were comparable; however, there were fewer patterning questions and reflection questions incorporated into the 2007 test.

Table B1

Differences between the 2007 and 2020 Grade three Island Numeracy Assessments (INA3+)

	2007 Edition	2020 Edition
	Number of Questions	
Patterning	4	13
Number Sense	20	19
Computational Fluency	13	16
Reflection	1	3

As represented through Table B1 and Figure B2, the INA (2007 version) addresses current mathematical concepts. In order to understand how collaborative group work can influence students' academics, one must first understand the mathematical concepts. The reason for this is because when an understanding of the mathematical concept is given, a greater awareness and understanding of how collaboration can be incorporated will be achieved. Below, an outline of the four main topics (patterning, number sense, computational fluency and reflection) in grade three mathematics will be provided.

Patterning. Patterning includes the ability to recognize, extend, and replicate numbers. It is considered fundamental to the understanding of mathematics (Baroody & Coslick, 2000; Steen, 1990). Prior to the 1990s, it was believed by educators that mathematical patterning should be left for older grades (Boaler, 2016). However, during the 1990s, the development of mathematical reasoning and problem-solving skills was introduced, acknowledging that younger children could develop more complex skills (Papic & Mulligan, 2007). Theorists have argued that developing strong patterning skills at a young age is critical to success later on in mathematics, as patterning may lead to mastering abstract cognitive skills such as identifying differences and similarities in objects (Papic& Mulligan, 2007); identifying abstract numbers (Papic & Mulligan, 2007); and generalizing the relationships between numbers (Threlfall, 2004). As students continue to develop knowledge within patterning, it allows them to grasp an understanding of relationships, as opposed to numbers being attached to one specific concrete idea (Kidd, 2014). When students are able to identify patterns and develop an understanding of how the pattern works, they are able to transfer their understanding, as opposed to only applying their newfound knowledge to the mathematical problem in their workbook. Therefore, this further exemplifies the importance of developing patterning skills at a young age which can be done through activities such as puzzles, sorting and games.

Number Sense. Number sense (NS) refers to the general understanding of numbers and operations (Reys, 1998). This can include using mathematical judgment to develop strategies and the ability to use numbers as a way of communicating ideas (Reys, 1998). However, NS can be a complex process as it involves using different components and showing an understanding of mathematical operations and their relationships (Yang & Li, 2008). Initial number sense conceptual development happens early in life, often before children even enter school (Dehaene, 2011). Initial NS development is based on understanding terms such as estimating and approximating, which then leads to an understanding of differences between quantities (Dehaene, 2011). As children continue to develop and enter a structured education system, it should seemingly follow that their understanding of number sense continues to develop. For instance, in a grade three setting, an understanding of number sense would be knowing that the values of 8 and 6 in 876543 represent one-hundred thousand and one thousand respectively (Yang & Li, 2008). Developing a strong number sense in early elementary is critical to the understanding of mathematics in older grade (Yang & Li, 2008).

Computational Fluency. Computational fluency is the ability to accurately and efficiently complete math calculations (National Council of Teachers of Mathematics, 2000). This can be observed when students are able to recall math facts automatically in a swift manner, as opposed to performing an algorithm (Vasilyeva, et al., 2015). An example of this would be the ability to recall that 4 x 3 = 12, as opposed to adding 4 + 4 + 4. When students are not proficient in computational fluency, they often struggle with advanced math skills (Wong & Evans, 2007). One explanation for this concept is that students may often be expected to have competency in this area as they grow older. Therefore, many teachers place less emphasis on instruction and the practice of basic math facts and instead work on building upon these basic math facts for more advanced problems (Vasilyeva, et al., 2015). For these reasons, it is imperative that teachers continue to incorporate computation fluency at a young age and that they are also aware of the need to review and potentially develop interventions if a weakness has been shown in this area.

Reflection. In the early 20^{th} century, Dewey (1933) introduced the concept of reflective thinking in regard to the learning process and claimed that “understanding happens when one acquires information and grasps how information relates to one another by constantly reflecting on the meaning of what is studied” (p.78). Reflection is a learning strategy where individuals self-monitor their own work, and through that, develop goals and plans which lead to a greater understanding of their own performance (Choi et al., 2017). This was recently confirmed by

Choi and colleagues (2017) who looked at the impact that self-reflection had on mathematical performance and levels of engagement. It was observed that when secondary students were required to complete online self-reflection, participation and confidence levels increased, which was then positively associated with a higher academic performance (Choi et al., 2017). From my observations, reflection is often a concept that needs to be guided in an elementary setting as the skill does not come naturally to students. In a whole class setting, this can look like guided writing activities, providing students with the opportunity create a video showing their understanding or reviewing an assignment after marking has occurred. Developing these habits at a young age can have a positive impact as it becomes a norm for them (Jackson, 2017).

Mathematics requires the ability to create connections, which is often observed when mathematical knowledge is taken and applied to real life situations (Nurhasanah et al., 2017). When students are able to make connections, it demonstrates a stronger understanding of the topic as they are now able to apply the knowledge they have learned (Hendriana, 2014). This is often seen through the use of word problems. The ability to make connections is one method of reflecting; however, it is also important that students are able to reflect on their own work in mathematics.

Social Learning in our Education Systems

Thus far, an understanding of theoretical views from seminal authors to modern day views has been provided, as well as, an explanation of how social learning can occur in mathematics. The following section will provide an outline of social learning in our education systems by identifying how our education system has developed in Canada, how curriculum has changed in Canada to now include core competencies, and how group work can be used as a form of social learning.

History of Group Work. As education has advanced, so have the ways in which formal instruction is given. The concept of formal instruction in Canada began in 1847 when normal schools were developed as a way of creating equality in education (Raptis, 2018). The term normale ("normal") schools was adapted from France's Ecoles Normales Superieurs to ensure that thoughts and values were unified by establishing a standard methodology of content and instruction in the educational system (Raptis, H, 2018). To grasp an understanding of the pivotal changes that have occurred, I created Figure B3 (see appendix for full version) as a way of summarizing the developments between 1847 and our current systems.

Throughout these historical changes, changes within curriculum itself and the methods utilized to teach the curriculum content were also observed. Since the first change in the 1950s, which incorporated the use of examinations in school, the BC Department of Education has developed changes and advances within the curriculum 10 times (BCTF, n.d.)[4].

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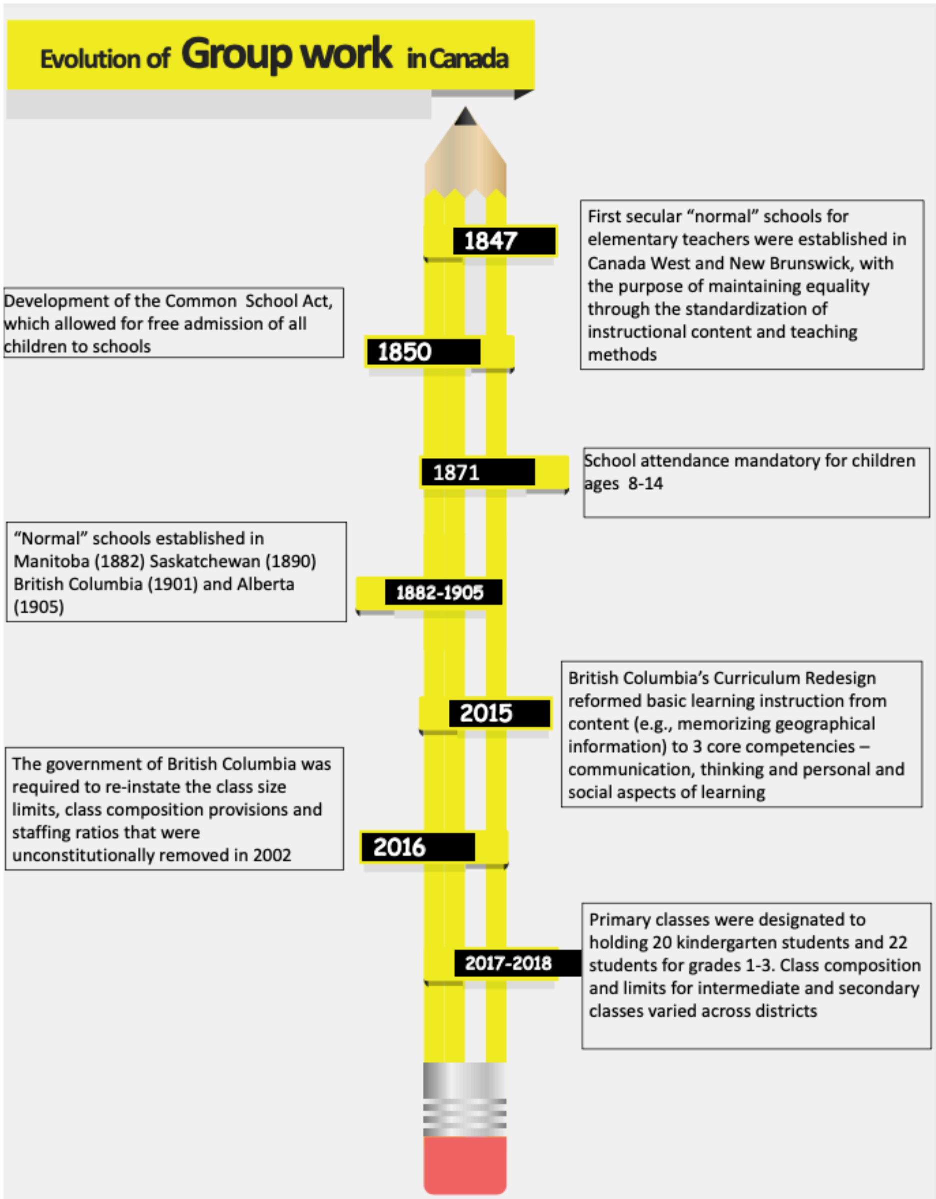
[4] Figure B4 was adapted from an online professional development document, which can be found on the BCTF website (https://bctf.ca/uploadedFiles/HistoryMuseum/Collections/HistoryArticles/CurriculumTimeline.pdf).

Figure B3

My Overview of the History of Group work in Canada



Note. For brevity and clarity of the figure, the relevant references are in a footnote below. A more detailed overview can be found in the appendix (see. Pg. 128) [5]

[5] References: 1847 (Raptis, H, 2018); 1850 (Robson, K. L, 2013); 1871 (Robson, K. L, 2013); 1882-1905 (Raptis, H, 2018); 2015 (BCTF, 2019); 2016 (BCTF, 2021); 2017-2018 (BC Curriculum, 2021).

In order to make sense of the multiple changes in BC Curriculum, Figure B4 provides a synopsis of the multiple changes in the BC Curriculum throughout the past 85 years. In Canada, recent changes in curriculum have shifted their focus to incorporated skills (i.e. communication), as opposed to focusing solely on academic outcomes. Developing these core competencies has been deemed essential within our education system according to the Ministry of Education. Therefore, to develop these skills, teachers must recognize that collaboration and group work need to be incorporated into learning.

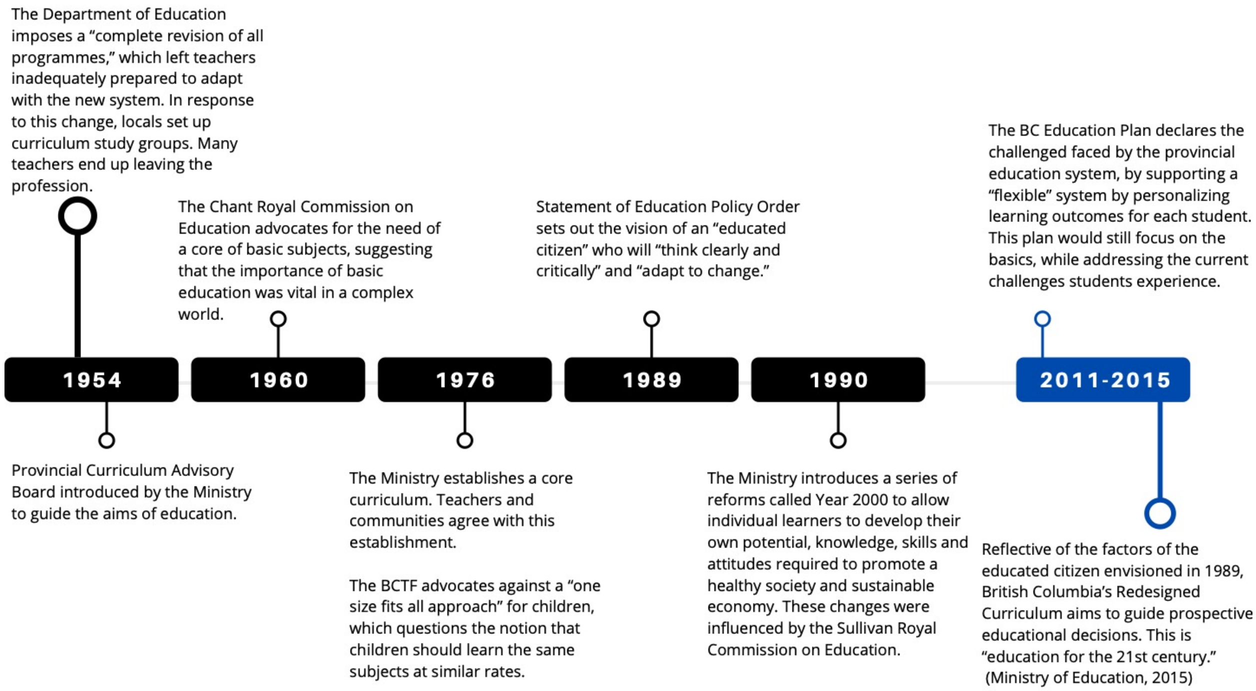
In addition to the incorporation of Core Competencies, another change within BC curriculum is added: acknowledgement for First Nations, Inuit and Métis in Canada. This is evident in a change to the Professional Standards for BC Teachers (BCTF, 2019) which now has an added standard nine to the former eight standards. This standard calls teachers to the following:

> "Educators respects and value the history of First Nations, Inuit and Métis in Canada and the impact of the past on the present and the future. Educators contribute towards truth, reconciliation and healing. Educators foster a deep understanding of ways of knowing and being, histories, and cultures of First Nations, Inuit and Métis" (BCTF, 2019, p. 5).

The importance of standard nine in this call to educators to critically examine their own belief systems and to envision a decolonized curriculum based on the First People's Principles of Learning (FNESC, 2020) is an important recent development for educators in the BC education system.

Figure B4

My Timeline of BC's Curriculum



I developed Figure B5 to show the skills (core competencies) that each province has deemed important. It is apparent that our country and provinces have recognized the need to adapt and change the methods, instruction, and content that students are learning. In BC, this has been done through three Core Competencies: personal and social, thinking and communication (BC Curriculum, 2020).

Figure B5

My Overview of Core Competencies in Canada

Note. All information listed is a general representation (K-12) as stated on provincial website.

Although each province has tailored their focus, while others have not yet included competencies, for the provinces that have, the one competency that is seen across numerous provinces is communication. Therefore, this next section will outline how social learning can be achieved through the use of communication. More specifically, the use of group work as a form of social learning will be identified through examining collaboration, academic-based groups and peer-based groups.

Social Learning. The incorporation of social learning provides an opportunity for collaboration. Collaboration, whether it be through peer-based groups or academic-based groups, can have a large influence on students. Students base their abilities on the most local frame of reference to them, which is often those they work closest with (Liem et al., 2015). When students feel supported by one another, it can greatly impact their learning because a student's effort is likely influenced directly by his or her peers behaviour, either through observation of peers' effort or through reinforcement by peers (Sorensen et al., 2017). Therefore, it is important to have collaboration in the classroom and to consider the best method when having students work with one another.

As previously mentioned, with the changes in BC Curriculum and the BCTC standards for teachers, two of the main focuses have been the introduction of core competencies (BC Curriculum, 2020) how teachers "critically examine their own biases, attitudes, beliefs, values and practices to facilitate change" (BC Curriculum, 2020) related to First Nations, Inuit and Métis in Canada. When social learning is incorporated into the classroom, students are provided with the opportunity to develop their core competencies (i.e. communication) while also including the First People's Principles of Learning (FPPL) (see Appendix B, Figure B6). The First People's Principles of Learning was developed by First Nations Education Steering Committee (FNESC) as a way of reflecting a respectful, holistic approaching to teaching and learning (FNESC, 2021). The desire of the FPPL is to identify common elements between current teaching practices that are also represented in the learning approaches for First Nations societies. The notion that learning is relational, experiential and involves the community are aspects that are represented in social learning and FPPL. When looking at the way in which social learning can be incorporated into a classroom, it is important to not only consider our own perspectives and understanding, but also those involved in our learning communities.

Collaboration in Groups. The way in which students work collaboratively can take on many different forms. An outline of two methods (academic-based and peer-based) will be provided in order to bring insight to the best method of structuring groups.

Academic-Based Groups. There are two methods that are often observed when academic-based groups are formed: tracking and ability-based groups. These terms are very closely related with the difference being that with tracking, students are unable to move from this group for a predetermined time (i.e. semester, term or year). In British Columbia, an example of tracking is when students are required to pick either Workplace Math 10 or Foundations of Mathematics 10. When students are placed in a lower learning track, they experience reduced learning opportunities and the possibility of a negative peer environment (Lavrijsen & Nicaise, 2006). New Zealand teachers report that teacher expectations often vary between the top and low streams of ability content (Anthony & Hunter, 2017). This meant that lower ability groups were being provided with low level content, as well as, fewer opportunities; whereas higher groups reported high-quality teaching which led to a sense of empowerment (Anthony & Hunter, 2017, p. 78).

The age at which students are placed in a learning track can have an impact on academic achievement. Lavrijsen and Nicaise (2016) examined the effect of the age at which tracking occurred in relation to student achievement through standardized student assessments that looked at both mathematics and literacy. It was determined that early tracking (elementary-aged students) had a negative effect on mean performance for literacy (Lavrijsen & Nicaise, 2016).

Furthermore, Hanushek and Woessmann (2006) found that "early tracking amplified the differential between weak and strong students" which further was confirmed by Lavrijsen and Nicaise, who also found that "tracking led to larger gaps between low and high performers" (Lavrijsen & Nicaise, 2006, p. 338). Lavrijsen and Nicaise expanded on the research conducted by Hanushek and Woessmann by increasing their sample size and also had a larger focus on the effect of tracking on both low and high achievers in comparison to the gap between individuals. Overall, they found that "early tracking does not seem to offer major benefits" (Lavrijsen & Nicaise, 2006, p. 345).

Given the research above, it shows that placing students in a determined group can have a negative effect on their academics in their current state, as well as, their future. Although placing students in different mathematical programs is not commonly seen in elementary schools, I have personally witnessed this. During my practicum, I became aware of elementary schools where students moved to a specific class for mathematics – resulting in advanced work, additional support or a decrease in exposure to content while still following the same curriculum. Furthermore, I have observed a version of tracking at School X in elementary grades. In previous years, students were provided with two 'groups' blocks where they would have additional support (educational assistance) for 40 minutes. Many teachers would use this as an opportunity for small group work where teachers openly admitted to grouping students based on their abilities – rarely changing groups and resulting in differed exposure to content.

Like tracking, academic-based groups, which are often referred to as ability groups, are a bit more flexible as students are not stuck in a specific course. Ability grouping has been observed to not just help students, but also assists teachers. Anthony and Hunter (2017) found that "teacher support for the dominant practice of ability grouping included claims that enabled them to target students with similar needs and abilities, and that the learners felt less intimidated when working with peers with similar abilities" (p. 81). Many Asian countries, including Singapore, continue to be extremely successful in mathematics and commonly use ability groups (Kaur, 2014). Furthermore, Soresen et al. (2017) looked at the influence peers have on both mathematical and reading abilities for students from grade three to eight in North Carolina. It was discovered that, overall, reading comprehension skills did benefit diverse levels of abilities; however, in math it was more advantageous for students to be with peers at a similar level (Soresen et al., 2017).

As identified in Table B2, grouping students based on their academic desires and abilities is still commonly used. According to the above literature, it does appear that tracking can be advantageous for a student's academic abilities; however, negative effects were observed in terms of a student's self-esteem and opportunity. Therefore, the question is raised as to whether a different way of structuring groups can benefit students both academically and socially, which is the reason peer-based groups is also explored as part of this study. It is important to acknowledge that the above literature explores multiple grades whereas the focus of this study was exclusively on grade three students.

Peer-Based Groups. Another method of group structuring is developing groups based on friendships. Depending on their age, students often spend more time with their peers and teachers than they do with their parents. Bissell-Havran and Loken (2009) found that friends have a similar value in regard to support as both parents and teachers do. This was further confirmed by Rice and colleagues (2013) when students in grades 5, 8 and 11 had a greater level of support from parents and friends, and teachers, and this had a positive influence on a student's self-efficacy. From my experiences, support can come in many forms some examples are: peers helping one another, parents providing encouragement and teachers taking the time to re-explain concepts.

European countries, specifically Finland, moved away from ability grouping and began grouping their students heterogeneously (Sahlberg, 2011). Although many Asian countries do still use ability groups, Japan has started to move away from this as well, as parents and teachers worried “ability grouping would have a strong negative impact on children's self-image, socialization patterns and academic competition” (Anthony & Hunter, 2017, p. 76). Singapore too has recognized the importance of self-concept in that their desired outcome is to produce students who are confident in their abilities (Singapore Ministry of Education, 2009).

As students’ friends have been recognized to have a considerable influence over each other’s academic performance, it raises the question of whether grouping students based on peer preference could have a positive impact (Veronneau et al., 2010). Rice and colleagues (2013) identified that students who appeared to have greater social support for math and science from parents, teachers, and friends, displayed better attitudes and an increased level of confidence in their abilities. Furthermore, Bissell-Havran and Loken (2009) found that students have a strong self-competence in mathematics when they feel great support from their friends.

Peers do “have a significant and lasting influence on a student’s learning and motivation” (Sorensen et al., 2017, p. 697). However, it may be important to further consider the impact on both high- and low-achieving students. For example, Verschaffel and colleagues (2007) found that students who were considered less academically skilled were able to use the same strategies as students who were academically stronger, but the more complex concepts that required advanced strategies were performed slower with less accuracy by the academically lower students. Therefore, this shows that lower-achieving students are able to perform the same tasks but may require additional time as concepts increase in difficulty. The impact of placing high-achieving students with low-achieving students was also examined by Steinberg and Monahan (2007). They acknowledge that high-achieving students can inspire and motivate their classmates, whether they are placed in ability groups or peer-based groups, however, low-achieving students often disrupt others' learning (Steinberg & Monahan, 2007).

The use of both academic-based groups and peer-based groups appears to still be inconclusive as there are positives and negatives for both structures. When considering peer-based groups the question of whether it is better to place low-achieving students with those who have similar abilities and focus or with high-achieving students who may have a positive influence is observed. Furthermore, it is important to also consider where educators want to place their focus – on supporting lower-achieving students or on providing advanced, more challenging work for high-achieving students. This next section will provide an understanding of what is currently being done around the world in terms of academic grouping, as well as, social learning.

Current use of Group Work Around the World. As already identified, grouping students is not a new concept to our education systems. Although, there is evidence that supports the use of both academic-based group and peer-based groups, one consistent theme is that providing opportunities for social learning, through group work, benefits students (e.g. Boaler, 2016; Hattie, 2017). Despite the fact that ability group may not be the most effective teaching strategy (Hattie, 2017), many countries, who have been considered academically strong in mathematics (according to the PISA), continue to use ability grouping. Table B2 gives us a representation of countries around the world who are currently using ability grouping.

Table B2

My examples of ability grouping by country

Ability Grouping by levels of streaming (i.e., grouping within schools) and tracking (i.e., grouping within classrooms; Chiu et al., 2017)

Country		PISA 2018 Mathematics Results
Singapore	Over 80% of students are grouped by ability in mathematics classes as confirmed by school principals (Organisation for Economic Co-operation and Development (OECD), 2013)	569
Japan	Schools that did not practice ability grouping in classes for 15-year-olds dropped by over 15 percent (OECD, 2013)	527
South Korea	Ability grouping has become more common than for some or no classes; schools that did not practice ability grouping in classes for 15-year-olds dropped by over 15 percent (OECD, 2013)	526
Poland	Ability grouping has become less common, with a 24 percent drop in the use of no ability grouping from 2003 to 2012 (OECD, 2013)	516
Canada	"The gap in academic achievement between streamed students from high and low socio-economic backgrounds increases with age, with the achievement gap in Mathematics doubling from age 7–11 to 12–15" (Caro, 2009)	512
Denmark	Ability grouping has become more common; schools no longer have some or all classes that to not practice ability grouping; schools that did not practice ability grouping in classes for 15-year-olds dropped by over 15 percent (OECD, 2013)	509
Finland	Streaming and tracking is performed until the end of Grade 9 (Aedo, et al. 2017)	507
Sweden	43% of secondary schools use ability grouping (Ramberg, 2015)	502
United Kingdom	71% of secondary students are taught using streams for mathematics (Francis et al., 2017); over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	502
Germany	The number of students who joined schools that practiced ability grouping in some or all classes increased between 2003 and 2012; schools that did not practice ability grouping in classes for 15-year-olds dropped by over 20 percent (OECD, 2013)	500
Ireland	Over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	500
New Zealand	Over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	494
Australia	98 percent of Australia's schools use some form of streaming (OECD, 2012); over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	491
Israel	Over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	463

Note. Countries have been listed in order according to their PISA ranking.

Conclusion

Group work asks students to socially construct knowledge in a space of collaboration. We learned from social constructivism, originally influenced by the work of Piaget (1963) and Vygotsky (1978), that knowledge and understanding can be developed jointly. This notion was also acknowledged by Bandura (1977) as the importance of social interactions as a way of development was recognized. In the same vein as these cognitive developmental theories on social interaction, it is also important to acknowledge the neurological developments and differences that influence how students learn together socially (e.g. executive functioning, domain specific development, influence of age, etc.). The ideas on students' neurological and social development have an influence on their academic achievement at school. However, development is not the only domain that influences academic achievement. How and within which structure children collaborate in a classroom has a huge influence on their academic learning.

Introducing the concept of collaboration into classroom aligns with the BC curriculum as BC identified that the current curriculum allows for instructional flexibility (BC Curriculum, 2020). This enables teachers to determine the best way to incorporate the learning of various concepts. Based on the prevailing literature, it appears that educators greatly vary in their method and reasoning for grouping as ability grouping, tracking, and groups defined by friendships (Sahlberg, 2011; Anthony & Kunger, 2017). Thus, there seems to be a need for further research in the way in which students are grouped to learn, and to learn math specifically. The objective of this study was to gain a further understanding of whether social learning can occur in mathematics, whether social learning in the form of group work is an effective teaching strategy, and whether greater academic benefits are observed between two different formats of social learning (academic-based and peer-based).

Chapter 3: Research Design and Methods

This chapter will discuss the design and methods used for this pilot study. More specifically an understanding of the hypotheses, an in-depth discussion of the methods used – specifically the instrument tool and a description of the sample will be provided.

Problem Statement

The development of mathematics is a critical skill to develop at a young age (Bjorklund et al., 2020). Kadosh and colleagues (2013) found that children's difficulties with mathematics can affect them beyond the classroom (e.g., future jobs, self-perception, confidence levels, etc.). One way to target specific mathematical difficulties is by providing an intervention (Kadosh, et al., 2013). When an intervention is applied, a strategy is put in place with a specifically desired outcome (Stacy et al., 2017)). Currently, many assistive programs that are digitally-based often use tablets (Stacy et al., 2017) or mobile applications (Outhwaite, 2020). However, there are also many approaches such as games and intervention strategies that teachers utilize in their day-to-day teaching to improve students' academic functioning in regard to mathematics.

While many teachers strive to meet the needs of their students, they are also restricted by the parameters placed by administrations (i.e., school districts, principals, etc.). However, teachers have the opportunity to control some aspects of their day-to-day teaching activities (e.g., creativity, execution of lessons, the format, etc.), which may affect the success of students. One such aspect is how they approach group work activities in the classroom.

Grouping students is a concept that has been in our educational systems for over 50 years (Boaler et al., 2000). However, it has often led to large inequalities between students, often culminating in strong students thriving and weak students growing weaker (Boaler et al., 2000). This gap is not currently as prominent in elementary schools; and therefore, a group work concept to learning math is often observed as common practice in teachers' classrooms (Agrawal et al., 2014). As discussed in Chapter 1 (see page 4), I often utilized group work in my teaching of math by placing students in groups, based on their abilities. I have also used a mixture of preference and ability-based groups for reading. In 2018, I started the year by doing a diagnostic assessment and placed students in reading groups based on their abilities as per the assessment. Throughout the duration of my study, I would work with each small group individually.

Later in the year, I allowed students to pick a story they were interested in, which then led to literature circles. From my observations, the understanding of the story and the students' ability to participate in discussion did not seem to be hindered by the type of group they were assigned. My observations are supported by literature that reveals that within-class grouping benefits students who were low achievers, as they received examples from higher achievers (Condron, 2008; Galloway-Bell, 2003; Elbaum et al., 1997). These observations made me question whether the impact of students' academic performance extends to mathematical achievement when placing them into structured groups and led to the current pilot study.

Research Objective(s)

Below are three hypotheses that are in accordance with the problem statement and align with the aims of this research project. A rationale for each hypothesis can be found in Chapter 1 (see pg. 18).

Research Hypotheses

Hypothesis One: An improvement would be observed between the pre-test score and post-test scores as a result of time.

Hypothesis Two: As measured by post-test INA mean scores, students in the peer-based group are expected to score higher than students in the academic-based group and the control group. Meanwhile, students in the academic-based group are expected to score below the peer-based group, yet higher than the control group.

Hypothesis Three: Students in the peer-based group will show a greater pre-test to post-test mean score improvement (as measured by an increase in the INA) than those who are placed in the academic-based group as a result of being able to work with peers who they are already comfortable with.

Design and Measures

Below, an explanation of the research design, variables, assessment tool, and adaptations to the assessment tool will be provided.

Research Design. The research design for this study is a 3 x 2 quasi-experimental intervention study. The data collected was solely quantitative. It is a quasi-experimental design as students were not randomly assigned. This format is often used in educational settings as it provides convenience for the researcher (Price & Jhangiani, 2015). For this study, the intervention was the structuring of groups (see Appendix A, Figure A2).

Independent variable(s). The aim of this research project was to determine if the structuring of groups affects academic progress for grade three students.

The independent variables is the structuring of groups (academic-based & peer-based).

Dependent variable. The dependent variable is the academic scores from the INA (pre- and post-test).

Control of extraneous variable(s). In order to minimize the effect of extraneous variables (e.g., classroom environment, instructions provided, differences in teachers, etc.) on the results, the following measures were taken:

1. Students performed INA in their own classrooms with their own teachers
2. Classrooms were silent for the duration of the assessment
3. Verbal instructions were provided at the beginning
4. If students asked for help from their teacher, the teacher reread the question, without providing additional help

Island Numeracy Assessment (INA). Introduced by the Island Numeracy Network, the Diagnostic Mathematics Assessment is a province-wide measure written by teachers in BC (Island Numeracy, 2020). It was developed to effectively assess students' content-knowledge, and design a tool that provided comprehensive, ongoing assessment to assist teachers with their instructional planning. The diagnostic aspect of the test contained multiple choice questions and short-answer word problems. There were eight research-based essential concepts that guided the assessment tool (National Council of Teachers Mathematics, 2014): I think about a math problem in my head first; I use math symbols and numbers to solve math problems; I use math tools, pictures, drawings and objects to solve math problems; I make a plan, called a strategy, to solve a math problem and discuss other learners' strategies tool; I use a strategy I've already used to solve another math problem; I check to see if my strategy and calculations are correct; I use what I already know about math to solve math problems; I try many times to understand and solve a math problem.

The Island Numeracy Network has also identified eight different resources that have influenced the design of their assessment: First Peoples Principles of Learning, Core

Competencies, BC Numeracy Network, Balanced Assessment, Growth Mindset, Inquiry, First Steps in Mathematics and British Columbia Association of Math Teachers (Island Numeracy, 2020).

Subtests. While there are no subtests for the INA, the Island Numeracy Network has developed assessment tools for students grades three to six. However, Island Numeracy Network states that "each assessment strand can be used more than once to check for growth and understanding" by repeating the assessment as a way of determining if there is an increase in knowledge and ability (Island Numeracy, 2020).

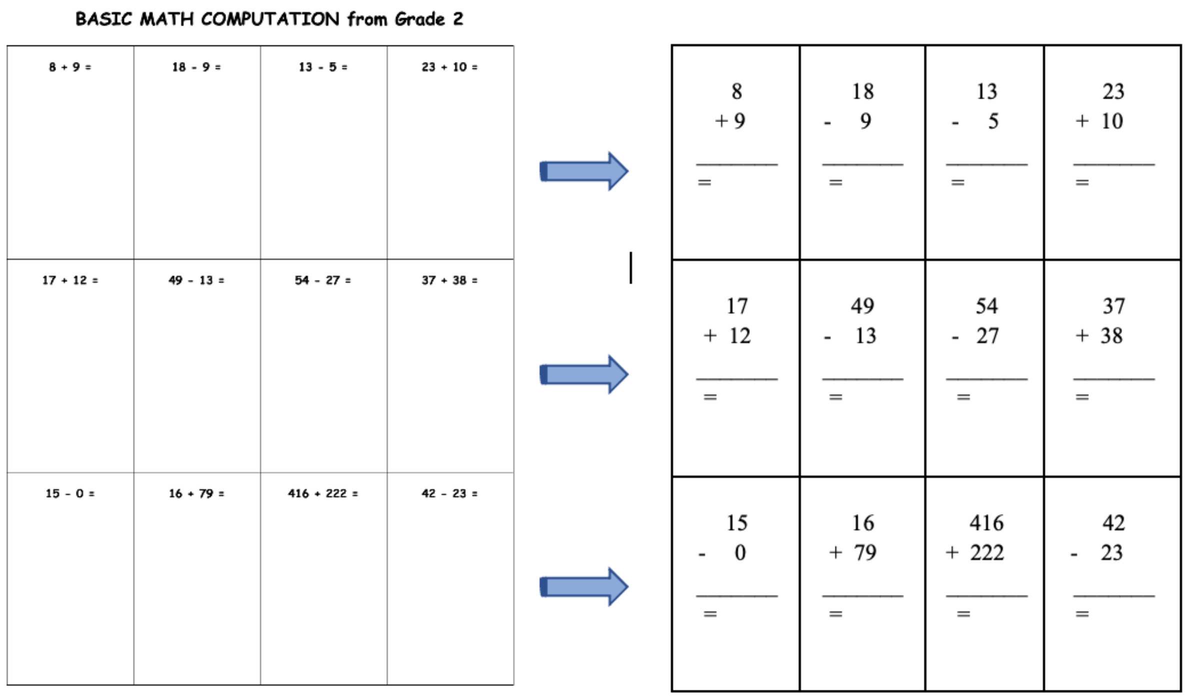
Adaptations. Below is an explanation for two areas that involved adaptations.

Time. As there were no instructions provided in regard to time, 40 minutes were provided for the diagnostic assessment. This is consistent with the time allocated for one block at School X. As the math computation section is much shorter, 20 minutes was provided. If students were absent at the time of the test, teachers did their best to provide the same format for students who completed the assessment; however, due to scheduling difficulties, students may have been required to complete the assessment in one sitting.

Formatting. The math computation aspect of INA was originally written with questions written horizontally, which can be seen on the first half of Figure C1. However, based on the advice of School X's Learning Support Coordinator, questions were rewritten to be vertical, resembling the format used for grade two classes at School X.

Figure C1

Sample of Math Computations of The Island Numeracy Assessment (INA)

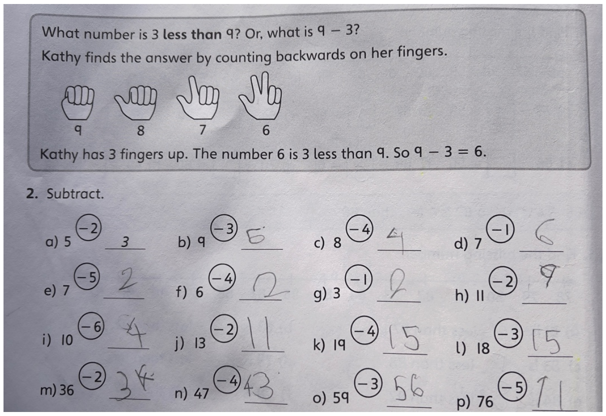


Note. Formatting was adapted from the INA's basic math computation (Island Numeracy, 2007) from horizontal to vertical (right).

JUMP Math. For mathematical instruction, School X uses the JUMP Math program from kindergarten to grade eight. JUMP Math was created by Dr. John Mighton in 1998 (JUMP Math, 2019). The Foundation of JUMP Math is built on a structured inquiry approach where students "discover and understand mathematical concepts by answering questions and working through challenges on their own, but they are also provided with a good deal of rigorous guidance and support from the teacher" (JUMP Math, 2019). This is achieved by the teacher asking a series of socratic questions, which are outlined in the JUMP Math Teacher Resource (JUMP Math, 2019). Figure C2-C4 are student examples that show how JUMP Math breaks down a concept and provides multiple opportunities for practice.

Figure C2

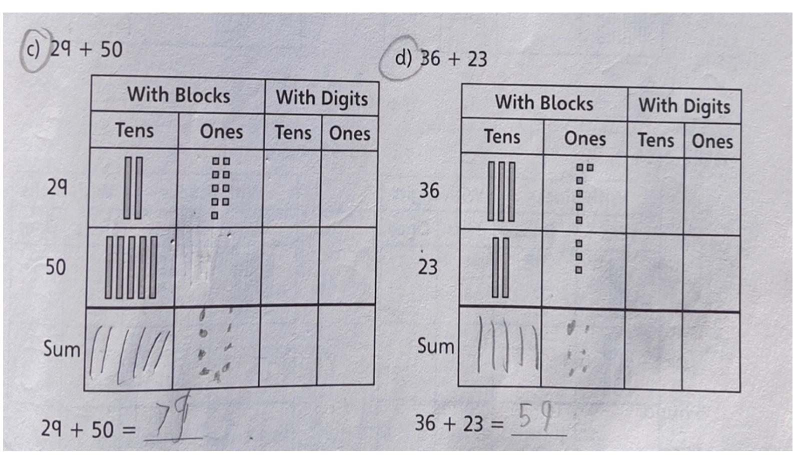
Student Work Sample - Subtraction



The above example identifies one of the methods students from all groups were taught. Using the JUMP math teacher manual, a lesson was first provided followed by the opportunity to practice this method for subtracting. Students in the peer-based and academic-based groups would have likely had the chance to practice this skill alone, as well as, with their group as JUMP Math provides multiple days to practice the same skill, slowly increasing in difficulty. Therefore, allowing students to gain confidence in their own abilities while also being provided with time to work with others where they could ask questions and work collaborative to solve problems.

Figure C3

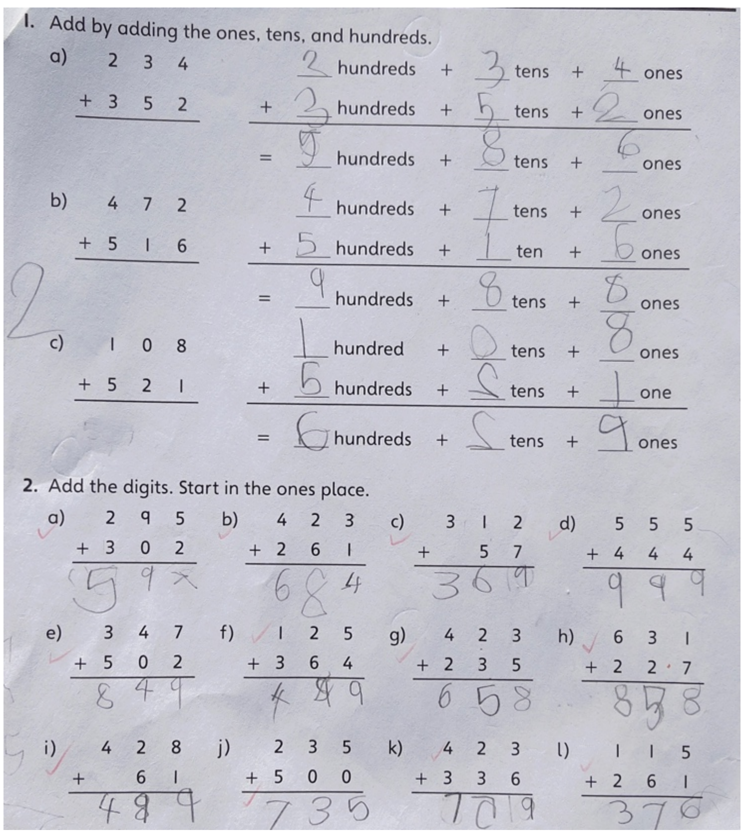
Student Work Sample – Number Sense



Using base ten blocks is one way JUMP Math increases a student's understanding of number sense. In the peer-based group, students were provided with the opportunity to use manipulatives (base ten blocks) to show their answers when working with each other. Other groups may have had the opportunity use base-ten blocks, as it was a recommendation by JUMP Math, however, as I did not oversee these groups, I am unable to report on this.

Figure C4

Student Work Sample – Addition



Similar to the first example, the above figure shows how JUMP Math scaffolds their learning as the concept of addition is broken down. The JUMP Math program has expanded on students' understanding of base ten blocks and related it to the concept of addition.

Reliability & Validity

Below, an understanding of both reliability and validity for the INA is provided.

Reliability. Reliability can be defined as the extent to which the findings can be replicated using the same methodology (Spencer et al., 2003). Therefore, in order for this to occur, the research instrument must be considered reliable (Joppe, 2000). As mentioned in Chapter 2, the INA was created with the BC curriculum in mind and was designed by current teachers with the intention of "providing teachers in [the] province with an effective assessment of student content-knowledge" (Island Numeracy Network, 2020). It was an assessment tool design by teachers for teachers with the desired outcome to inform instructional decisions and create an awareness for students in regard to their own understanding of mathematics (Island Numeracy Network, 2020).

In order to assess the reliability of an instrument, one would normally refer to the r-factor. When an r-factor is provided, it means a statistical analysis has been conducted to determine the variation an instrument produced when the tool is used repeatedly (Bannigan &Watson, 2009). However, as this was an instrument designed to be accessible and free, it has not gone through rigorous testing, such as other mathematical assessments like the Kaufman Test of Educational Achievement (Pearson, 2021). Since an r-factor is not provided, the reliability will be determined by experience. The INA has been used at School X since 2010 as a way of assessing students' academic knowledge at the beginning of the year. Through observations from my colleagues, as well as myself, each year similar results are yielded; and therefore, we believe it can be considered a reliable measurement tool given the scope of this study.

Validity. Once an instrument has been determined reliable, one needs to look at how well the tool measures what you are wanting to measure (Bannigan &Watson, 2009). The INA provides a resource that outlines which categories (e.g., patterning, number sense) each question falls into. As demonstrated by Figure B2 (see Appendix B), the INA assesses students based on topics that are currently part of the BC Curriculum. Construct validity, which is often used in educational settings, determines how well the test is actually measuring the content it claims to be (Statistics How To, 2020). All of the questions on the INA directly relate to concepts learned in grade three; therefore, given the scope of this study, construct validity is considered to be relatively assumed.

Sampling - Participants

72 participants were chosen as a result of convenience sampling. The sample could be regarded as convenient because I used students from the current grade I was teaching at School X (Taherdoost, H, 2016).

Table C1

Distribution of Demographic Characteristics Across Sex and Age According to Birth Month

	N = 71	%
Sex		
Male	39	54.9
Female	32	45.1
Age		
January	6	8.45
February	4	5.63
March	11	15.49
April	3	4.23
May	6	8.45
June	4	5.63
July	6	8.45
August	2	2.82
September	5	7.04

October	11	15.49
November	5	7.04
December	4	5.63
Born in 2009/2010	4	5.63

Of the 72 participants, one was excluded due to a significant time away from school, therefore, resulting in a total of 71 participants. Table C1 shows basic demographic information which was used as moderators (i.e., sex and age according to birth month). A complete breakdown of the sample can be viewed in Appendix H.

Participants (*N* = 71) were recruited via convenience sampling – a form of non-probability sampling where individuals are recruited based on the feasibility of the researchers needs; this includes accessibility, geographical proximity, availability at a given time and the willingness for individuals to participate (Dörnyei, 2007). Although convenience sampling is a common practice for pilot studies, it is important to acknowledge that the data may not be fully credible as it often only considers a small demographic (Marshall, 1996; Elfil & Negida, 2017).

From my experience and observations, all elementary class sizes at School X range from between 24 and 27 students. In the elementary grades (K-7) at School X, student placement is determined with the assistance of the previous grade's teachers. From my experience as a grade three teacher, the administration requires us to consider specific learning challenges (IEP & LP), as well as, our observation of a student's level of academic performance, communication skills and social skills. At School X, teachers do not assign specific students to a teacher, but instead strive to create four equal classes per grade.

According to school records, 78% of the current grade three students have been at School X since kindergarten. Through my observations, having a large number of students together since kindergarten has created a strong sense of community. School X is a commuter school, which has created an environment where parents are heavily involved. A commuter school means that students travel to attend school unlike the public-school system where students attend based on their location. Based on these two aspects, the understanding of peer influence becomes increasingly interesting. Table C2 identifies the demographics of the current grade three students at School X.

Table C2

Distribution of Demographic Characteristics Across Ethnicity, Language and Family Structure

	N = 71	%
Ethnicity		
Caucasian / White	16	19.72
East Asian (Chinese, Japanese, Korean, Taiwanese)	40	40.8
African / Black	3	4.23
South Asian (e.g., Indian, Pakistani, Sri Lankan)	2	2.82
Other	12	16.9
Mother tongue		
English	41	57.7
Chinese	5	7.04
Korean	5	7.04
Mandarin	15	21.1
Other	5	7.04
Second Language Spoken at Home		
No	40	56.3

Yes	31	43.7
Number of Children in The Family		
One	5	7.04
Two	43	60.6
Three	16	22.5
Four	6	8.45
Six	1	1.41
Position of the Child		
Oldest	24	33.8
Middle	8	11.3
Youngest	34	47.9
No siblings	5	7.04
Marital Status of Parents		
Married	64	90.1
Divorced	7	9.86
Education of Mother		
High School	3	4.23
Certificate Program or Some Post-Secondary	57	80.3
Master's Degree	8	11.3
Did Not State	3	4.23
Education of Father		
High School	7	9.86
Certificate Program or Some Post-Secondary	41	57.7
Master's Degree	13	18.3
PhD	5	7.04
Did Not State	5	7.04
Individualized Education Plan (IEP) or Learning Plan (LP)		

None	67	94.4
Learning Plan	3	4.23
IEP	1	1.41

Academic and behavioural challenges are both acknowledged through Individualized Educational Plans (IEPs) and Learning Plans (LPs). LPs can be considered the step before an IEP where the classroom teacher and learning support coordinator have recognized a need, whether it be academic or social. At this point, students do not have a formal diagnoses or IEP; however, it has been recognized that the student requires additional support beyond what is normally provided in the classroom.

The number of students with IEPs and LPS is fairly low in comparison to other schools. The British Columbia Teacher Federation (BCTF) stated in 2019 that the number of students with special needs was approximately 10% of the total population (BCTF, 2019). At the commencement of this study, the three classes that took part in the study had three students who were on IEPs. However, consent was only given for one of these students to participate in the study. Based on this information, .05% of students are on IEPs which is far below the average in BC.

Collection Methods

This study was designed as a quasi-experiment intervention pilot study to consider how the structuring of groups affects the academic performance for grade three students. Quasi-experiment research received its name as it resembles experimental research, but cannot truly be considered that, as participants are not randomly assigned (Chiang et al., 2015). Quasi-experiment studies, often conducted in field work, can be categorized into three types:

interrupted time series designs, designs with control groups and designs without control groups (Schweizer et al., 2016). This study falls into the *designs with control group* and is indicative of a field study as it was conducted in my classroom. One of the greatest advantages of using quasi-experimental designs is that they are less expensive and less time consuming, which were two of the reasons this design was chosen (Schweizer et al., 2016).

A quantitative approach was taken with descriptive statistics collected through a parent questionnaire as this allowed to test the hypothesis and determine whether there were any confounding variables (Johnson & Onwuegbuzie, 2004). As previously stated, the participants came from a very select area (School X), due to convenience; therefore, only sex and age were considered.

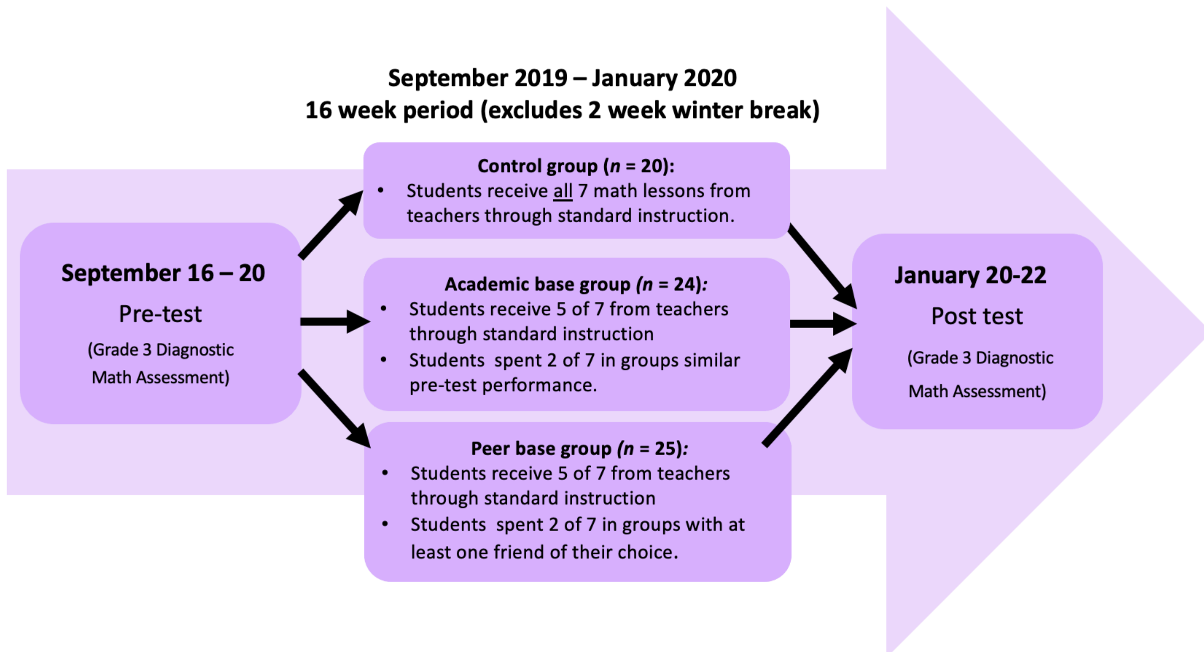
Procedure

Once parental consent had been given, prior to the start of the grade three mathematics program, all participants completed the Island Numeracy Network Assessment, which was used as a pre-test. Upon completion of the pre-test, an intervention was applied, which was the structuring of groups. The peer-based and academic-based groups designated two 40-minute blocks per week where students worked in their groups. Control Group completed their math work as directed by their teacher. Figure A2, which was originally shown in Chapter 1, but has been placed below for convenience, displays the phases of research, and shows the procedure for study.

All three classes (peer-based, academic based and control) were taught and instructed in regard to their mathematics as directed by the JUMP Math teacher guide as this provided consistency amongst all groups.

Figure A2

Experimental Procedure



After completion of 3.1, the first of two math books that grade three students completed throughout the year, all three groups repeated the Island Numeracy Network Assessment as a post-test. All participants completed the post-test between January 20-22, 2020. If a student was not present during the time that the exam was given, s/he completed it upon return to school.

Academic-Based Group. The academic-based group was designated as the academic-based group as the teacher's previous methodology included math groups that were determined by ability. The academic-based groups were determined by the results of the pre-test. Students in the academic groups were placed in equal groups as an outcome of their academic performance.

Peer-Based Group. Using a self-reporting format, students in the peer-based group were provided with the opportunity to write down three friends they would like to be grouped with for math class. In grade two, they completed this for classroom placements, so it was a format they were familiar with. From these results, students were placed into four groups that were as equal in numbers as possible. It is important to acknowledge that due to the way in which students are placed in classes at School X, I knew that everyone had at least one friend from the year before. During this study, there were no new students; however, if this had been the case, the new student could have rotated through groups as a way of getting to know students and then report to the teacher which group they felt most comfortable in.

If this study was to be replicated, it may be advantageous to start the study later in the year and have the teacher create groups based on her observations of friendships. Research has found that peers are naturally drawn to others with similar academic and behavioural attributes (Wang et al., 2019). Furthermore, when children naturally go through a selection process of deciding who they want to be friends with, one of the influential factors can be academic achievement (Greeman et al., 2017). Therefore, with enough time and observation, teachers would be able to create groups that represent similar interests and attitudes towards learning. If the teacher was to make peer-based groups strictly off his/her observations, as opposed to self-reporting, it would also account for students who may struggle socially.

Statistical Techniques

In order to obtain accurate data, all tests were marked using the answer key provided by the Island Numeracy Network. All word problems were marked according to the rubric, which was provided as part of the answer key.

Data was analyzed using the Statistical Packages for Social Sciences (SPSS; IBM, 2009). As the study had a measure that was repeated (INA) and included within-subject factors (sex and age), a repeated measures Analysis of Variance (ANOVA) was used to determine if there was a difference between groups and whether sex or age was a moderator.

Ethical Consideration

I received approval for Human Ethic Review (Appendix F) from the Research Ethics Board at Trinity Western University. As students are all minors, parents provided consent for participation. Alderson and Morrow (2004) acknowledge that conducting research involving potential risks versus the likely benefits should always require ethical considerations. In order to obtain data for this research project, students performed two conventional additional assessments to confirm that potential risk were low to none. Below is a brief description of ethical considerations.

1. *Safe learning environments* were provided as all students performed INA within the comforts of their own classrooms.
2. *Participation* was voluntary and participants could remove themselves at any time with no consequences.
3. *Privacy* was insured as all responses were collected and stored in a password protected computer file, in a locked filing cabinet, or off-campus.
4. *Anonymity* was provided as participants were assigned an identification number, which was only known to the researcher. The participants' names will not be published.

Conclusion

Through this chapter, a summary of the research design, sampling, data collection, procedure, data analysis and ethical considerations was provided. Furthermore, a great understanding of INA, with an explanation for adaptations, was given. The next chapter will provide an analysis of the data that was collected.

Chapter 4: Data Analysis

Data collection in this study as collected prior to and post the intervention stage by means of the Vancouver Island Diagnostic Mathematics Assessment (DMA), will be referred to as the Vancouver Island Numeracy Assessment (INA) throughout this paper. This assessment tool was used as a pre-test and was followed by placement of participants into three groups: academic-based, peer-based and a control group. The results of the initial INA tool were then compared to the results 16 weeks later upon the completion of JUMP Math 3.1, where participants completed the INA a second time (Appendix A; Figure A2). I used IBM SPSS Statistics - Version 26 for my statistical analysis, which is commonly used to analyze data in educational settings (IBM Corp, 2019; Mut et al., 2019).

Preliminary Analysis

Each participant was assigned an identification number ranging from 1-71. As part of parental consent, parents also provided answers to a questionnaire which addressed demographics and learning needs. Results from both the INA and parent questionnaire were recorded with the student's identification number in an Excel Spreadsheet, and were then entered into SPSS. In order to do this, data was coded for all categorical variables (i.e., sex and age).

Slavin and Smith (2009) explain the relationship between sample sizes and effect sizes in education, where small sample sizes are often used to qualify whether it is a topic that should be replicated or extended to further see the interactions. The authors also acknowledge that the information gained in small studies should not be ignored as they can pave the way for larger studies. However, the findings of larger, more controlled studies should be given greater weight (Slavin & Smith, 2009). Given that this study consisted of a relatively small, homogenous sample (N = 71), sex and age were the only moderators taken into consideration.

Descriptive statistics

71 grade three students from School X participated in this study. Chapter 3 (see pg. 61) provides an in-depth overview of the sample. Below a detailed outline of both sex and gender are provided as they were further considered as moderators based on previous literature.

Sex. As a self-reporting system was used for peer-based groups, it resulted in peer-based groups being divided by sex as girls and boys chose to be partnered with peers of similar sex. Erturan and Jansen (2015) found that gender had an influence in mathematics as a difference was observed in emotional experience (i.e., math anxiety) between sexes. More specifically, it was discovered that girls in grades three to eight displayed a higher level of math anxiety (Devine et al., 2012), which then led to decrease in mathematical abilities (Erturan & Jansen, 2015). This raised the question of how sex might impact the results, as explained in Chapter 2 (see pg. 32). Figure D1 depicts the differences in sex across the three groups.

Table D1

Distribution of Sex Across Groups

	Male	Female
Peer-Based Group	13	12
Academic-Based Group	23	11
Control Group	9	14
Total	35	37

Age. Through classroom observations, I noticed that children who were born between September and December were often lagging behind their older peers academically. This has been an informal observation; and therefore, I have always wondered whether it carries merit. Upon further research, I recognized that working memory, which is an executive functioning skill, is one of strongest links between academic successes in mathematics (Bull and Lee, 2014; Lan et al., 2011; Andersson, 2007). The reason for this is that children who have a greater working memory capacity have a larger storage area for cognitive resource, and therefore, can contain the information that is required when carrying out mathematical problems that involve previously learned information and multiple steps (Swanson and Kim, 2007). Additionally, theory of mind, which is the ability to understand mental states (e.g., thoughts, desires, emotions), can also greatly benefit mathematical abilities (Frye et al.,1995, Wellman and Liu, 2004). Already knowing that executive function and theory of mind skills develop as children get older (Devine & Hughes, 2013; Blaire & Raven, 2015), Cantin and colleagues (2016) explored if a difference would be observed when a specific range was considered (7–10-year-olds). It was found that executive function abilities were influenced by age and that age suggestion had a direct correlation with mathematics and theory of mind performance (Catin et al., 2016).

Therefore, based on my personal observations and research described in Chapter 2 and the above explanations, this warranted further investigation. As this study only contains one grade, a division needed to be created to determine if the age of students within a grade level could affect mathematical abilities. The groups were differentiated in the following manner: oldest (January-April), middle (May-August) and youngest (September-December; see Figure D2).

Table D2

Distribution of Students by Birth Month and Group

	Birth Month	Birthdate Group	Oldest to Youngest by Group
	Number of Students		
Born in 2009/2010	3	3	1
January	6	24	2
February	4		
March	11		
April	3		
May	6	18	3
June	4		
July	6		
August	2		
September	5	25	4
October	11		
November	5		
December	4		

Note. 1 = oldest; 4 = youngest

Distribution of data

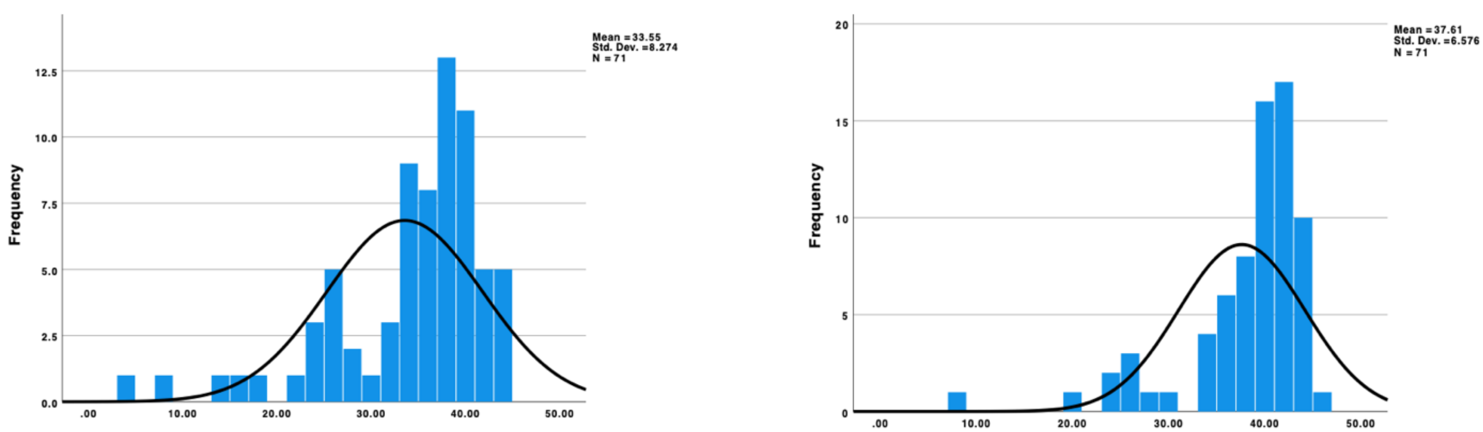
A preliminary assessment of the data, using two methods (Skewness and Kurtosis and Histograms), was done to determine if the data set was normally distributed.

Skewness and Kurtosis. In statistics, skewness and kurtosis are used to test for normality (Laerd Statics, 2013). Scores with a skewness less than -1 or greater than 1 indicate a highly skewed distribution, which was the case for both the pre-test and post-test (see Appendix D).

Histograms. Histograms, which show the distribution of the data, can also be used to gain an understanding of the spread of scores. Figure D3 further confirm that the data are not evenly disturbed (pre-test: SD = 8.27; Shapiro-Wilk's test ($p > .05$); post-test: SD = 6.58; Shapiro-Wilk's test ($p > .05$) (Laerd Statics, 2013). Therefore, the data cannot be considered representative of the general public (see further discussion in Chapter 5). However, this is likely due to the small sample size and outliers (Laerd Statics, 2013). It is important to note that outliers were not removed for the pre-test and post test scores as the sample size was already small and was intended to be representative of all students (e.g., students with learning and behavioural needs).

Figure D3

Histogram of Pre-Test and Post-Test Scores



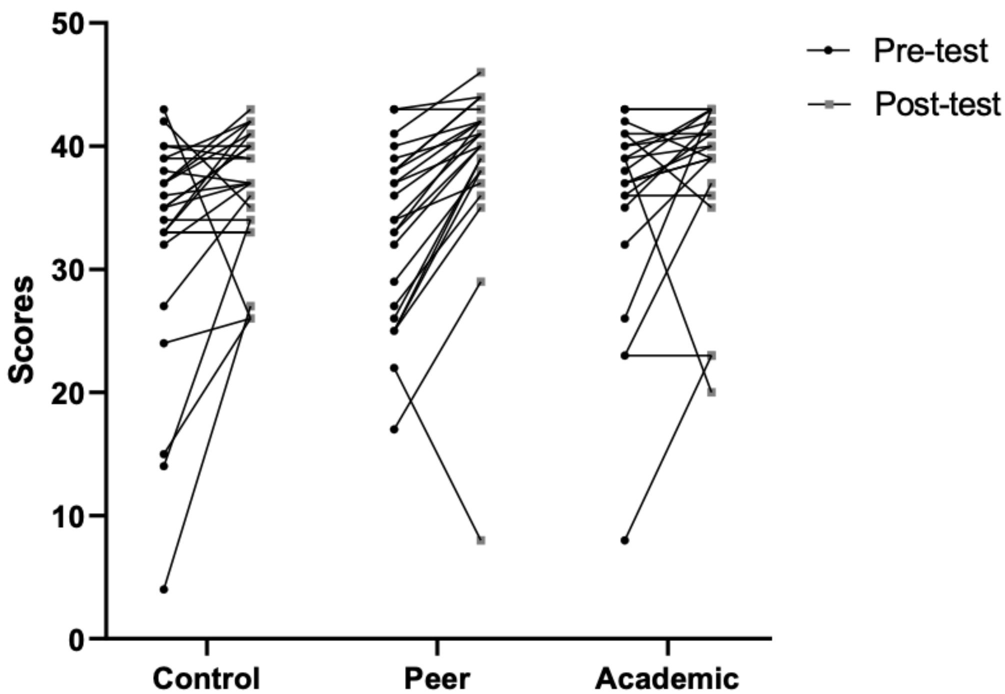
Note. Pre-test scores (left side); Post-test scores (right side)

Results

Past research identified that when students are exposed to mathematical content, their academic abilities often improve over time (Rotherham & Willingbham, 2009; Cai et al., 2016). However, additional factors can also impact a student's academic achievements. Noted during my observations, teachers tend to have their own beliefs and practice; however, one practice that is often used is group work (Slavin, 2014). I observed various teaching methods, even within group work; and this warranted further investigation. This next section will provide an understanding of the results from my quantitative data.

Figure D5

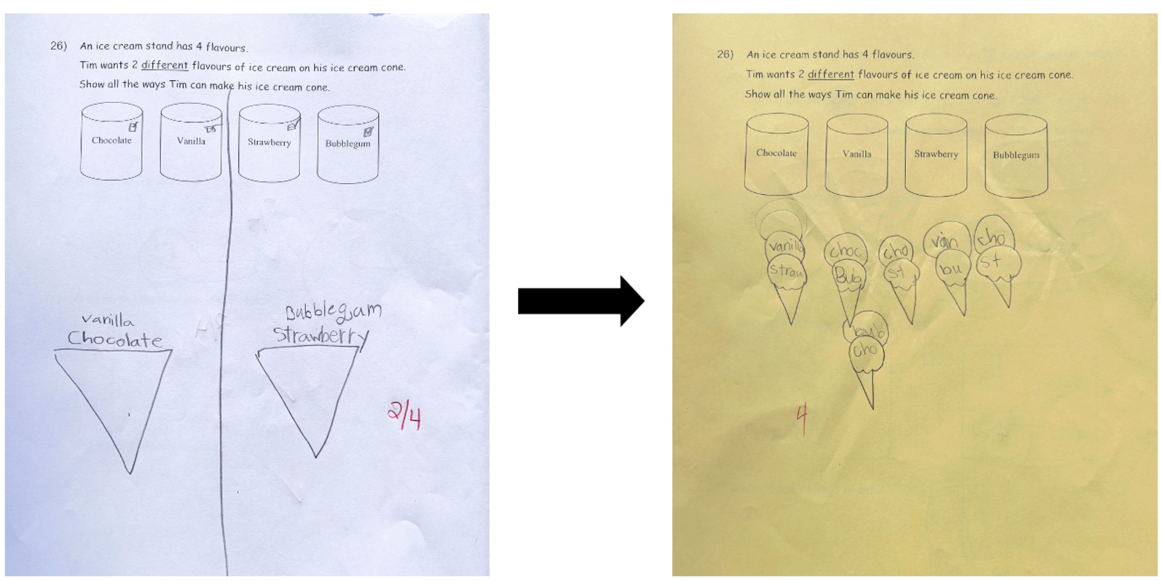
Comparison of academic performance across groups



A two-way repeated measure, ANOVA, was conducted to determine whether group structure (peer-based groups and academic-based groups) was the reason for a significant increase in academic improvement. Results of the students' residuals indicated violation of normality, as assessed by the Shapiro-Wilk test ($p > .05$). While a main effect of time was observed, $F(1, 71) = 22.9$, $p < .05$, $\eta^2 = 0.26$, there was no main effect of group $F(2, 71) = 0.73$, p .669 , $\eta^2 = 0.97$, nor an interaction between these factors $F(2, 71) = 0.95$, $p = .835$, $\eta^2 = 0.03$. The Vancouver Island Numeracy Network does not provide an indication for general results to be used as a comparison. However, an overall increase was observed from the pre-test (M = 33.55, SD = 8.27, $p < .001$) to the post-test (M = 37.61, SD = -6.58, $p < .001$). Figure D5 provides a visual comparison of academic performance across groups as measured by the INA. Although no statistical significance was found, a larger increase in mean test scores was observed for the peer-based as represented by a consistent large consistent upwards trend when compared to the academic-based groups compared to the control group (see Figure D5).

Figure D6

Student Sample from INA



In addition to statistical analysis, Figure D6 provides an example of student work that demonstrates the academic improvements that occurred between times. An improvement in academic performance can be observed between the pretest (in white) and post-test (in yellow).

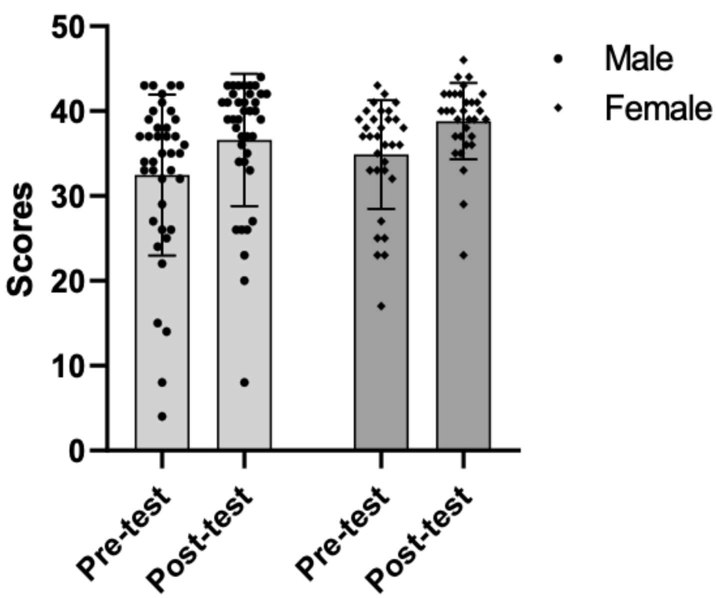
As depicted on the post-test a greater understanding of how to approach a word problem (e.g. drawing picture examples) can be observed. This sample was taken from the same participant in the peer-based group.

Moderators – Sex and Age

No significance differences emerged between groups when sex and age were considered. Additional analyses were conducted to determine whether sex or age were moderators. As explained in Chapter 2 (see pg. 31-32), sex and age may have had an impact on mathematical academic achievements.

Figure D7

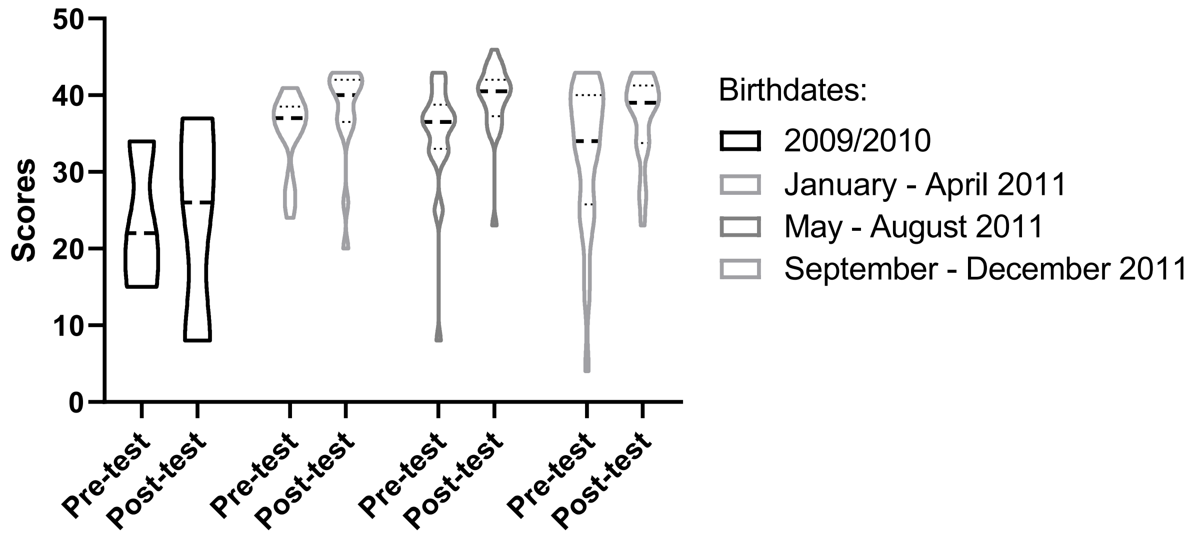
Mean comparison of post-test scores across sex



Age. Based on previous research, as described in Chapter 2 and classroom observations, age was also considered as a moderator. Despite there being no statistical significance (p = .84), Figure D8 does show a variance between birthdates.

Figure D8

Mean scores across age group



Students who were held back either due to academic challenges in school or because of parent decisions, scored lower when compared to their peers. Furthermore, students who would be considered oldest (January-April birthdates) in their grade had a small variance amongst their scores and also more consistent outcomes. Additionally, the greatest difference between pre- and post-test scores, as determined by Figure D8, was observed by those who were youngest in the grade (September-December birthdates). Although my study did not produce significant results when birthdates were used as a moderator, the results do show some difference and therefore warrant further investigation if this study was to be replicated.

Conclusion

The purpose of this study was to determine if the structuring of groups affects academic progress in math for grade three students at School X. Based on the data collected, as represented through the figures and charts above, students showed a statistically significant increase in mathematical abilities over time. However, there was no significant difference between groups (peer-based, academic-based and control group). When sex and age were considered as moderators, again, no significant differences emerged. Suggested explanations for these results will be discussed in Chapter 5.

Chapter 5: Findings and Conclusion

This study explored how group structuring may be utilized to improve grade three students' academic abilities in mathematics. A three by two design was used given our three groups (peer-based, academic-based and control) and measures at two time periods (pre-test and post-test). To determine whether the structuring of groups was impacted by students' academic math performance, students completed a pre- and post-test in the form of the Vancouver Island Numeracy Assessment (INA). Over a 16-week period, students were assigned to the three groups (peer-based, academic-based and control). The data was then analyzed using a two-way repeated measure, ANOVA, to determine if there was a difference among these factors. This chapter will summarize the methodology, discuss findings of this study with its implications, and deliberate plausible explanations for these findings.

At the beginning of this study, the desired approach was to interview teachers and students to gain insight into their experiences. By incorporating qualitative data, it would provide additional knowledge as to their enjoyment and perspective on mathematics when placed in specific groups. However, due to the shutdown of in-class instruction because of the COVID-19 pandemic, I was unable to conduct this aspect of my research. Therefore, instead, I have included my own personal reflection as a way of providing further insight. Furthermore, I have listed a set of questions that would have been asked to both students and teachers which would have increased the richness of this study by providing a story to coincide with the data. If this study was to be replicated, it would be important to incorporate qualitative data such as the list of questions I have provided (see pg. 98).

Research Questions and Hypotheses

The main research question of this study is: How does the structuring of groups affect academic progress in math for grade three students in an independent school in Surrey, BC? I hypothesized that there would be a statistical significance between the two main effects (time and groups) and one interaction (time x groups). This research question builds upon existing research of, amongst others; domain specific development, repeated exposure to mathematical concepts, and collaboration, while also bringing to light some of the current challenges in education systems.

Hypothesis 1. In order to gain an understanding of whether the structuring of groups affected academic progress, it was first important to understand whether students were improving academically over time. Therefore, my first hypothesis was that grade three students would show an improvement following the post-test, as measured by the Island Numeracy Assessment (INA). This was proven true as the data showed a statistically significant improvement between times: pre-test (M = 33.55, SD = 8.27, p < .001) to the post-test (M = 37.61, SD = -6.58, p < .001).

This hypothesis has also been confirmed in existing research through the concept of repeated exposure, which was achieved as a result of the JUMP Math Program. Throughout the duration of the study (16-weeks), students participated in a mathematical program (JUMP Math), which was already used at School X. JUMP Math was not used as an intervention, but simply as a tool. The JUMP Math program was already used at School X (K-7); and therefore, this was continued throughout the duration of the study. It also provided consistency in regard to how information was taught, and practice received. JUMP Math provided scaffolding, continuous assessment, and an innovative instructional approach (JUMP Math, 2011). It has been acknowledged that when students receive effective, repeated exposure to teaching concepts, academic improvements are observed (Barr & Dreeben, 1983; Entwisle & Alexander, 1999; Pianta et al., 2008).

Personal Reflection. An improvement in mathematical abilities over time is also consistent with past research on domain specific development (i.e., a type of neurological development), as it has been shown that mathematics (e.g., numerical operations) assists in the development of overall mathematical knowledge (Fuchs et al., 2010). Therefore, based on the results of my study and the support of previous research, when students are provided with the opportunity to have relevant teaching, exposure to grade level content, including a focus on development specific mathematic skills, an improvement in academic achievement will be observed.

As a classroom teacher, I have observed this natural progression and expected that students would improve over time in their mathematical learning irrespective of the groupings. However, I wanted to ensure this was true because it would first show that all students had gained knowledge and it would determine if there were any underlying factors (i.e. sex and age) for the whole group before analyzing the difference between the groups (peer-based and academic-based).

Hypothesis 2. Secondly, I wanted to determine which group would demonstrate the strongest mathematical abilities. As identified in Table B2, ability grouping is still used throughout the world; however, the evidence supporting the positive influence that peers have when they feel supported socially has also been recognized. Therefore, I hypothesized that post-test INA scores would be highest for participants in the peer-based group, followed by the academic-based group and then the lowest scores for the control group. However, no statistical significance was found in this respect. Despite the lack of significance, Figure D5 shows the peer-based group as having the greatest number of students increase over time. Therefore, in order to yield the results, I hypothesized, a larger sample size and increased duration of the study is likely necessary.

Recent research has shown that social learning has a positive impact on a student's academic performance (Bakar et al., 2010; Korir & Kipkemboi, 2014; DeLay et al., 2016). Additional factors such as level of engagement, praise received, and opportunities received have also been observed to positively impact a student (Boaler, 2016). Knowing that students improve academically over time when placed in groups, the desired outcome of this hypothesis was to determine how this occurred – whether being able to work with peers at the same academic level as oneself or being able to work with students who one would feel comfortable with had a greater influence.

When a student is engaged in his/her work, a greater development of skills and knowledge are gained (Wang & Holcombe, 2010). However, our education system often views improvement through more work. Students who do not complete their work in class or after class are required to complete it for homework; however, homework that is not attached to meaning is not interpreted as worthwhile, and therefore, has been observed to have no effect on achievement (Challenge Success, 2012). Learning needs to have meaning, and one way this can be achieved is through social learning, as it provides the opportunity to work collaboratively towards a common goal (Boaler, 2016). When this happens, student engagement can increase as peer relationships grow stronger (Coco & Kim, 2017).

Another area that has been observed to have a positive impact on learning is the use of praise (Boaler, 2016). When students are in a cohesive classroom where they feel supported by their environment, it has had positive influence on student learning (Hattie, 2017). By incorporating the use of peer-based groups into my study, the desired outcome was to provide a learning opportunity that was positive, as opposed to one in which students may have realized they had been labeled by their academic performance.

When students are placed in ability groups, those in the lower groups are often not exposed to the same content as those in higher ability groups (Anthony & Hunter, 2017). When looking at the difference between high ability and low ability groups, Boaler (2016) found that the difference was in their ability to use number flexibly – high-achieving students were able to interact with the numbers flexibly and conceptually; however, the low-achieving students were only able to recall methods they had used in class. Unfortunately, low-achieving students are often just given more drills and practice with limited opportunity to see math as a creative, visual, connected subject (Boaler, 2016).

Given that the research does appear to support the use of peer-based, below an explanation as to why my study did not produce this, as well as, an observation of those in the peer-base group will be provided.

Personal Reflection. I had also previously put students in math groups based on their abilities, so although I was not able to observe the academic-based group for this study, I will reflect on my own observations from the previous teaching year. When I placed students in academic-based groups for mathematics, I was able to reduce work, re-teach concepts, provide additional manipulatives or challenge them with more difficult questions, and this was something I had become accustomed to. This essentially made my job as a teacher easier as I was able to target the specific needs of a group instead of address them all individually. Interestingly, students did not seem opposed to being placed in ability-based groups – almost as if this is something they had become used to.

Fast-forward to my study where my class was the peer-based group. The reasoning behind using my class for the peer-based group was based on previous conversations with my colleagues that led me to believe they would not want students working with their friends. Some of the comments I had previously heard were:

1. “Rachel – I don’t know how you let your students work in their own groups so often, I could never do that”
2. “When I don’t put students in girl/boy order, they never listen”
3. “I think I might have to assign seating spots at the carpet because my students always talk to each other when they are allowed to be with their friends”

Knowing that I was allowing students to work with their friends, I already had the preconceived idea that I would spend half the time working through behavioural issues. Furthermore, I had additional hesitations because I had never allowed students to work with their friends for a challenging academic subject on a repeated basis. I felt that many other teachers would likely have similar reservations. However, the results of putting students in peer-based groups was shocking. Students immediately became excited about the idea of groups, which in previous years had not always been a source of excitement. For students who appeared to be anxious about mathematics (three specific students who showed this continuously at the beginning of the year), I noticed that their facial expressions began to change when I said that today was “group time”. However, it wasn’t just their facial expressions that changed, but their stature as well – I could visibly see their bodies relax. The positive effects were not only visible for those who appeared anxious towards mathematics. I found I was repeatedly being asked at the beginning of each math block if today was a “groups” day with excitement in their voices. I even had to change our schedule to state which days were groups days, as disappointment became a regular reaction when I said “no”.

I originally stated that one of my main concerns was whether students would fall behind academically when placed in peer-based groups. However, from my observations, that was not the case as an increased level of support and help was observed. When they faced challenges, they felt comfortable asking their friends for help and were able to complete the same work. I even overheard a student who was struggling with a concept say to his friend, “Wait … can you slow down and explain that to me? – I don’t really get how we got the answer.”

Although my data did not produce results that showed peer-based groups outperformed academic-based groups academically, the social benefits that I was observed resulted in keeping peer-based groups for weekly Google meets when we were required to go online as a result of COVID-19. Although one could hypothesize many reasons as to why there were not significant results, I believe that some of the dominant factors were: length of study, sample size and opportunity for group work. As a result of our school timetable, students in the intervention groups were only provided with two blocks (out of a possible five) of group work per week. This equates to a maximum of 80 minutes per week while still having a minimum of three and a half hours of whole class teaching. Furthermore, if the study was to be longer in length, the relationships formed in the peer-based groups could strength as this study was performed at the beginning of the year. Despite students knowing one another as they had all attended grade two together (no new students were in the peer-based group), their classroom composition was different, and students had just come back from summer holidays. As this was a pilot study, these factors could not be avoided, however, they may have impacted my results.

Hypothesis 3. Lastly, the study focused on a possible relation between groups and time. Therefore, I hypothesized that students in the peer-based group would show a greater pre-test to post-test mean score improvement (as measured by an increase in the INA) than those who were placed in the academic-based group as a result of being able to work with peers who they were already comfortable with.

Again, no statistical significance was found. My hypothesis was prompted by my own observations and questions regarding best practice as an elementary school teacher. The concept of ability grouping at School X was common in both language arts and mathematics; and knowing that BC requires students to be in a track for mathematics in Grade 10, I started to question whether this was the best method for structuring groups.

For the last few decades (1990 - 2020), countries around the world have used tracking and ability grouping (Steenbergen-Hu et al., 2016). Throughout the last three decades, ability grouping has been observed in Hong Kong, Singapore (Slavin, 1990), United States (Loveless, 2009), United Kingdom (Spina, 2019), Turkey (Bölükbaş &, Gür, 2020), Australia (Macqueen, 2012), New Zealand (Anthony & Hunter, 2017), Greece, Brazil and Japan (Spina, 2019).

In 2018, the Trends in International Mathematics and Science Study (TIMSS); which provides an international comparison for mathematics and sciences; ranked Singapore, Hong Kong, and Korea as the top three countries in the advanced benchmark for mathematics (TIMSS, 2015). The TIMSS provides an understanding of how students in countries perform in comparison to each other using a benchmark system (advanced, high, intermediate, low) (TIMSS, 2019). Although ability grouping has been observed during the last two decades, during this time, a shift has also occurred as there has been a realization that ability grouping can negatively affect students socially (Anthony & Hunter, 2017). Singapore, (Liu et al., 2005), Hong Kong (Marsh et al., 2000; Lit et al., 2001), and Korea (Hwang, 2014) have also observed negative self-esteem and negative self-perception in academics when ability grouping and tracking are used. The realization that ability grouping could also impact students in a negative way socially, led to an in-depth look at how education systems should be structured in Singapore, Hong Kong and Korea (Lit et al., 2001). It is evident that practices are changing in regard to the best way to structure groups; however, despite evidence showing the need to focus on academic self-concept and self-esteem, the countries with the highest performance in school-based mathematics still display some form of ability grouping (Lit et al., 2001; Liu et al., 2005; Hwang, 2014).

Although the results of my study did not lead to a significant finding when the groups were compared, the topic of tracking and ability grouping is still being studied. Given that my sample was small (N= 71) and looked at a specific demographic (i.e., one school, similar socio-economic status), a larger sample that is more representative of the general population may yield different results. Similar to my second hypothesis, I think that one of the major factors resulting in no significant findings is the length of the study. As a grade three teacher, I typically spend the first month of school focusing on creating a healthy classroom environment. Since this study was only four months long, students did not have a long time to develop strong relationships and were, as a result, likely still determining whether they felt my classroom was a safe, welcoming environment for the first half of the study. From my observations, students at School X generally come from supportive families. I have witnessed excitement to see their parents, a willingness between siblings to help one another and parents who are supportive of each other. Therefore, students may have already had a higher level of self-esteem resulting in a smaller difference observed when provided with peer support. However, it is important to acknowledge that these are just my own anecdotal observations and are not supported by any data provided by School X.

Personal Reflection. My reflection of this hypothesis is similar to hypothesis 2 as both address my hypothesis of peer-based groups outperforming academic-based groups. Although my study did not yield any significant results, as I observed the positive way students were interacting and the excitement that was evident, I wondered whether the direct impact on their academics would have been significant if the study had been longer in duration. If students have already attached a negative emotion to a subject, a five-month study likely would not be long enough to reverse the adverse emotions they had developed. Developing confidence takes time, and although I did observe a change in students' reactions and emotions, not all group work times were successful. Challenges still occurred, such as frustration when struggling to understand a concept, momentary defeat when their friends finished their work before them (despite helping them after) and the volume level in the classroom, as our blocks of group work resulted in a louder classroom than when individual work was completed. Therefore, perhaps if this study was able to occur over multiple years using the same students, different results would be observed.

Moderators

In addition to the three hypotheses, two moderators were considered: sex and age. From my classroom observations and based on current research, two additional questions were proposed:

1. Does the age of a student in the group impact their mathematical performance?
2. Does the composition of sex (i.e., same sex vs. mixed) in groups impact a grade three student's mathematical performance?

Age has been observed to affect academic performance as it has been linked to maturity level (Rodrigeuz, 2016; Thoren et al., 2016). Yesil, Dagli and Jones (2012) found that children who had been delayed in enrolment for kindergarten, meaning they started school a year behind their peers, had stronger mathematics skills than those who were enrolled on-time. Similar results were reported by Crawford, Dearden, and Greaves (2014) who considered the impact of chronological age on academic performance by looking at the difference when all grade one students were given an assessment on the same date versus giving the students the assessment once they reached a certain age. It was discovered that students who were born later in the year (e.g., June) performed worse when given the assessment at the same time as their peers (Crawford, Dearden & Greaves, 2014). This suggests that the month students are born in, and the resulting age difference, may influence academic results. Oddly, our results contrast these findings perhaps due to a small sample size or the number of students who did not have an IEP at the time of the study, but by the end were given a diagnosis ($n = 5$).

Sex was also considered as a moderator based on the fact that the peer-based groups were divided by sex, whereas the academic-based groups were mixed. Math has been observed as a male dominant subject, furthering the importance of questioning whether sex could be a moderator (Tiedemann, 2002; Boaler, 2016). Hyde and colleagues (2009) also analyzed gender differences amongst grades five to eight students in Beijing (n = 73,318), where it was discovered that there was no gender difference for grade five students and a relatively small gender difference for grade eight students resulting in females scoring slightly higher, but once again showing a great variance in scores for male students. However, when researchers consider the impact of a student’s environment as a factor in their performance, one of the greatest influences was whether the student felt a sense of belonging (Good, Rattan & Dweck, 2012).

Women stated that they often did not feel this sense of belonging and that as a result they pursued fewer math courses (Good, Rattan &Dweck, 2012). For example, the number of women who took PhD courses in mathematics between 2004 and 2013 fell from 34% to 27% (Velez, Maxwell & Rose, 2013). From my observations, grade three students do not show large academic difference in mathematics. When considering self-esteem and levels of anxiety, typically girls have shown an increased level when it comes to tests, however, the male participants in this study at times also showed a high level. Therefore, perhaps the age at which sex is consider as a moderator should be further considered as oppose to just looking at the differences between sex.

Summary of Findings

The aim of this study was to determine whether the structuring of groups impacted grade three students' mathematical academic abilities. Results revealed that grade three students' mathematical abilities improved over the 16 weeks of the implemented program. However, when determining whether or not a difference was found between groups (peer-based, academic-based and control), no difference was found in academic achievement. Possible suggestions of limitations are discussed below. Sex and age were considered for moderators; however, neither showed a significant influence on academic abilities.

Implications of Research

The concept of collaboration in teaching and learning is not new (Vygotsky, 1978). However, there is limited research on the impact of collaboration based on group structuring (Webb, 1991; Steenbergen-Hu et al., 2016). Even though this study did not really yield significant results on academic performance, the results (as per Figure D5) did show a consistent upwards trend for those in the peer-based group. Furthermore, a significant difference was observed between pre- and post-test for both the academic-based and peer-based groups, suggesting that providing students with the opportunity to work in groups may have a positive influence on the level of academic success.

As described in the considerations for future research (see pg. 95), peers have an influence on one another. Social learning and group work can be considered interrelated as both entail discussion, collaboration, and interaction when people are placed in groups (Vygotsky, 1962). Such elements encourage communication among group members and foster learning (Hill et al., 2009). Thus, an educational environment that entails discussion, collaboration, and interaction, especially in group work, promotes social learning. This further emphasizes the importance of considering the way in which we structure groups in elementary school.

Another aspect that warrants further investigation is the impact of chronological age (as represented in Figure D8). Figure D8 depicted that students who were younger typically started further behind their peers. This suggests that considerations should be made as to how that gap can be decreased.

Limitations

While this study outlines the importance of group structure on learning outcomes, my research design poses some limitations. I collected data via convenience sampling, which means that the results obtained are not representative of the population. Leon, Davis and Kramer (2011) discuss how pilot studies enhance feasibility and provide insight into modifications for a refined research design and methodology. This study could be viewed as a type of pilot study. It differs from most pilot studies in that it is not representative of the larger population (Marshall, 1996), but as it provides initial information from a non-representative smaller sample, it could give valuable information on which future studies could be based. Leon, Davis and Kramer (2011) note that this type of pilot study is still useful as it provides a springboard for initial steps towards larger studies (Van Teijlingen & Hundley, 2001), which could seek to replicate and extend this work in public schools with a diverse sample across the province.

Furthermore, teacher involvement was an additional limitation that may have affected academic performance. The delivery of information, consistent follow-ups with the JUMP Math teacher guide and time spent on mathematics could have varied. The impact that teachers have has been identified by Hattie (2003), who acknowledges that teachers have one of the largest influences (30%) on academic achievement. Currently, primary school teachers in many countries are not specialized in mathematics and are often seen teaching multiple subjects (Voskoglou, 2019). This is a common theme observed at School X and had the potential to impact my study, as some teachers may not have had the proper mathematics background needed for teaching mathematics.

In addition, the INA was used out of convenience and accessibility. The INA and current BC grade three Mathematics curriculum were compared to see if the INA would provide an accurate representation of grade three students' mathematical academic knowledge. Although it does accurately assess the current grade three mathematics curriculum, there are some limitations. I used the version created in 2007 before the second version was developed. One aspect that was not acknowledged in the 2007 is the inclusion of the First People's Principles of Learning (FPPL). However, according to the Island Numeracy Network, who created both the 2007 and the updated 2020 version, consideration of the FPPL was used to inform the assessment design (Island Numeracy Network, 2020). This speaks much to the authors' attempt at decolonizing the assessment to "in part" take into consideration indigenous knowledge systems. Though by no means comprehensive, it is a start. Although the mathematics curriculum has not changed, the way in which questions were presented and the lack of reflective questions does provide some limitations as to the generalizability of the results. Secondly, there were no Form A and Form B. If a test has a Form A and Form B, it means that there are two versions which ask similar questions, but do not repeat the same questions (Pearson Assessment, n.d.). When there is no Form A/B, it becomes a limitation because of test-retest reliability, as students were already exposed to the information (Aldridge et al., 2017). Thirdly, the INA is not a Level B test. Level B tests (e.g., Woodcock-Johnson Achievement, Kauffman Test of Educational Achievement, KeyMath Diagnostic, etc.) require specific training for administration and interpretation, and are often used to test whole group achievement or as a way of screening students (University of British Columbia, n.d.). These assessment tools have multiple forms, have a reliability factor and provide a more in-depth understanding of a student's academic achievement. However, I was advised not to use a Level B test because it may have impacted future results if a student was tested for a learning disability. Time would become a factor and there was a financial investment as each student required his or her own copy.

Considerations for Future Research

Below, a discussion will be provided as to considerations for future research, such as the difference between grades and potential differences, when comparing private/independent schools and public schools.

Multiple Grades. As this study only considers the impact of grade three students, future work could examine the differences between primary (kindergarten - three) and intermediate grades (four – seven). Elementary school students are still unable to pick their courses at this time and are required to follow the requirements of their specific school. Notably, students who do not acquire the correct mathematical background in primary school find it difficult to perform well in mathematics in later years (Voskoglou, 2019). Therefore, perhaps it is important to not only consider group structuring, but also, the impact on age. Peer influence has been thoroughly examined, including the effects of peer influence on general academic achievement (Coleman, 1961; Pellegrini, 1992; Johnson, 2000). However, it appears that age impacts the severity of the influence as the impact of peer relationships change as students become older (Thoren et al., 2016).

Private/independent schools and Public schools. Students who attend private or independent schools are at a specific institution based on parental decisions. At School X, 70 percent of the current students have attended there since kindergarten. However, School District 36, which is the catchment for School X, was unable to provide similar data on tracking student attendance. In BC, students are required to attend the school closest to where they live (referred to as "their catchment"). From anecdotal conversations with teachers, who are part of School District 36, it became evident that they are unaware of how many students they will have in their class on the first day of school. This leads me to believe that there is a greater variance when considering the number of returning students when compared to School X. Friendships are developed through time spent together (Narr et al., 2017); therefore, if a child is required to move schools, it can be argued that the level of friendships they develop may not be as deep. By looking at the difference between private/independent schools and public schools, it may yield a greater understanding of how peers influence each other based on the premise of time spent together.

Socio-Economic Status. Researchers around the world have found a significant correlation between Socio Economic Status (SES) and academic achievement, where students with lower SES backgrounds are influenced negatively (e.g., National Center for Education

Statistics, 2013; Organization for Economic Co-operation and Development, 2009). The understanding that SES is an influential factor in education began with two reports: one in United States (Coleman, 1966) and one in the United Kingdom (Peaker, 1971). These reports stated that "family background was more important than school factors in determining children's educational achievement" (Buchmann, 2002, p. 166), and that the SES of a family could explain more than half of the difference in student achievement across schools (Leithwood et al., 2004). The relationship between SES and academic achievement has also been observed in Canada where lower SES appears to result in lower academic achievement (Sirin, 2005; Berkowitz et al., 2016). Recently in Canada (Ontario), it was observed that SES accounted for 17 percent of the variance within districts (219 elementary schools across 24 districts) when mathematic achievement was considered (Patten, 2019).

Furthermore, recent evidence suggests that peer relationships can be affected by SES (Damen et al., 2021). This is largely due to the number of immigrant families, as immigrant families who have a lower SES tend to interact with those of the same ethnicity compared to immigrant families who have a higher SES who tend to be in contact with peers who are native to the area (Briggs, 2007; Vervoort & Dagevos, 2011). Therefore, this warrants further investigation as both ethnicity and SES appear to have an influence on the academic achievements of students.

Incorporation of Qualitative Data. As previously explained in the introduction, post study interviews for both students and teachers would have added to a greater understanding of the impact peer-based groups and academic-based groups have. Therefore, below (Figure E1), you will find suggested questions to ask both students and teachers if this study was to be replicated. Providing qualitative data through questions, such as the ones above, would add to the richness of the study as an increased understanding of how group work impacts students and teachers would be provided.

Table E1

Suggested Question for Teachers and Students

	Teachers	*Students*
Do you like math?	✔	✔
Do you feel you have a good understanding of mathematical concepts	✔	✔
Name one difference you experienced when working with your group vs working alone		✔
Describe one emotion you felt when it was a "groups day" for math	✔	✔
Describe your observations when students worked in groups	✔	
What challenges did you face because of teaching students in groups	✔	
Did you like working in groups – why or why not?	✔	✔

Conclusion

The purpose of this study was to explore the impact that group structuring has on mathematical academic achievement in BC. In summary, learning from others through group work has been a foundational part of our academic systems (Vygotsky, 1978). Ability grouping is a concept that has been used world-wide (Sahlberg, 2011) as a way of increasing academic achievement; however, it has recently been observed that grouping based on academics may not be the best instructional method (Anthony & Hunger, 2017). Therefore, this warranted further investigation by examining the structuring of groups and how they affected academic progress in math for grade three students. Using the INA as a pre-test, students were placed in two groups: academic-based and peer-based, with a control group used as comparison. After 16 weeks, students partook in a post-test (INA) to see if there was a difference between times, a difference in post-test scores, or a difference between groups in academic achievement.

Chapter One provides context for the study and elaborates on the importance of the topic. Chapter Two contains a literature review, which outlines philosophical views, neurological development, and previous research regarding groups. Chapter Three presents a discussion on the INA as it outlines the design and analysis of data. Chapter Four presents the results after statistical analysis was performed. Chapter Five provides a discussion of the results as presented in Chapter Four, as well as, limitations and suggestions for future research. The main conclusion of this study is that results show a significant difference in performance when time was considered (pre-test – post-test results); however, there was no significance difference in post-test scores or between groups.

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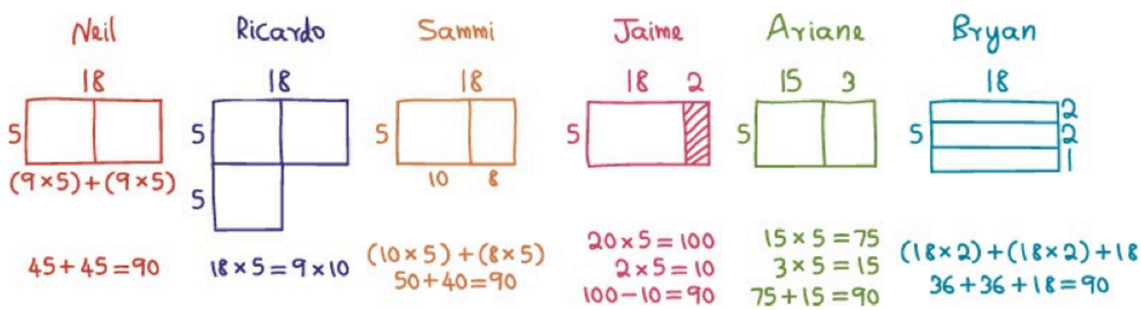
Appendix A – Chapter 1: Background and Rationale

Figure A1

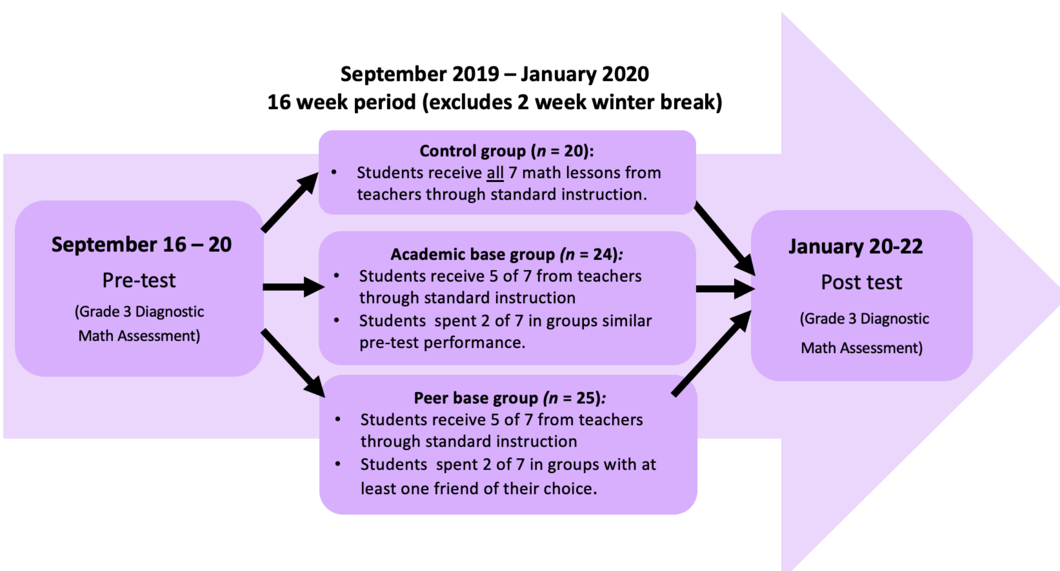
Visual Solutions to 18 x 5[6]



Note. Example is taken from work conducted by Jo Boaler (2016)

Figure A2

Experimental Procedure



Note. Each math lesson approximated to 40 minutes of instruction and work time.

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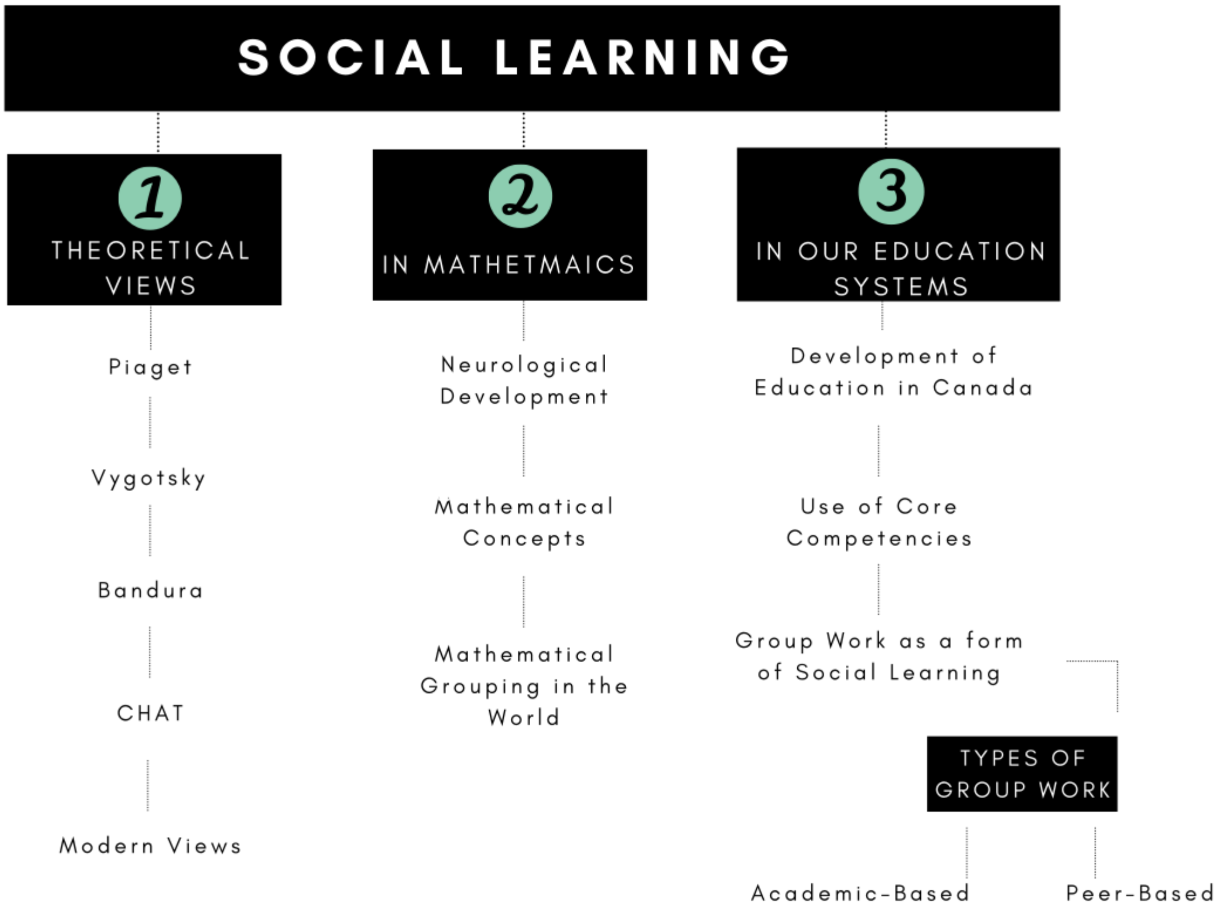
[6] Boaler, 2016, pg. 59

Appendix B – Chapter 2: Literature Review

Figure B1

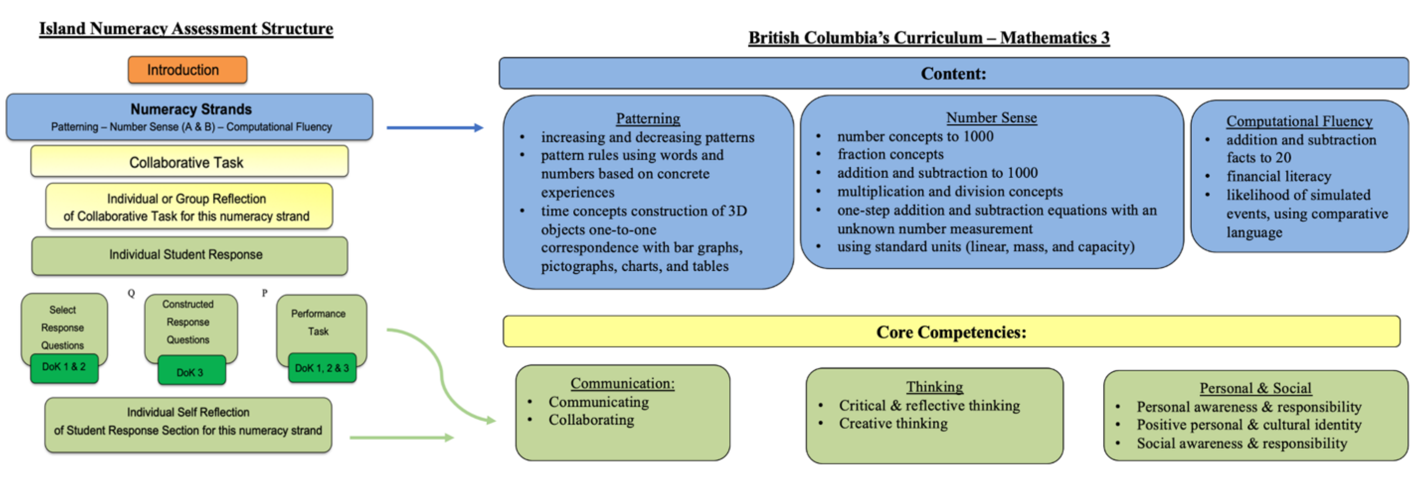
Visual Representation of Chapter 2 Topics



Note. My Chapter 2 is divided into three main topics (as observed through the number boxes) with various subtopics.

Figure B2

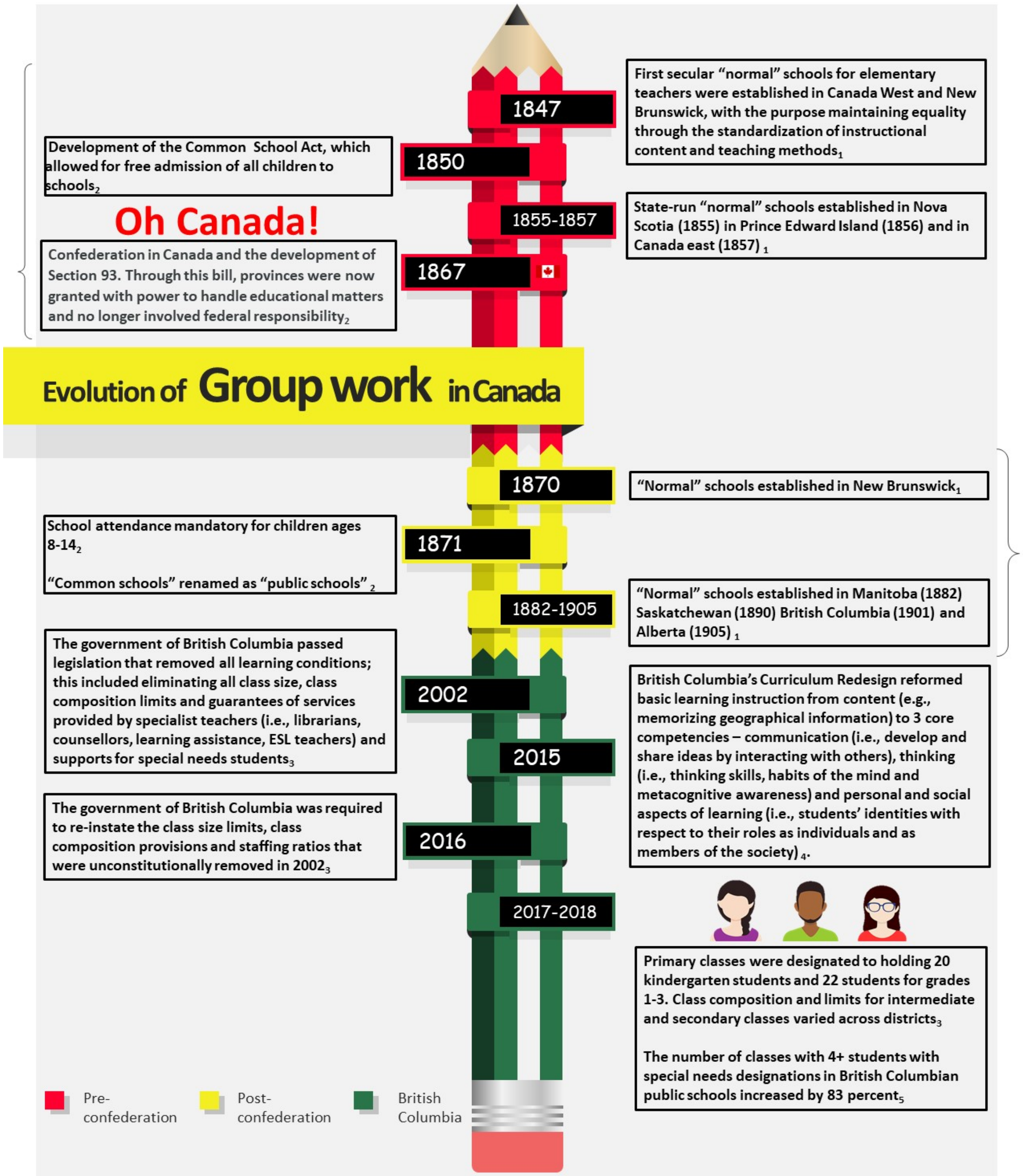
Comparison of BC Mathematic Curriculum and Island Numeracy Assessment (INA)



Note. The INA covers the content and core competences for BC Curriculum as identified through colour coding.

Figure B3

My Detailed Overview of the History of Group work in Canada



Note. "Normal" schools were adapted from France's "Ecoles Normales Superieurs," to ensure that thoughts and values were unified by establishing a standard methodology of content and instruction in the educational system[1]

1.Raptis, H. (2018). The Canadian landscape: Provinces, territories, nations and identities. In T. M. Christou (Ed.), The curriculum history of Canadian teacher education (1st ed., pp.7-22). Routledge. https://doi.org/10.4324/9781315411378
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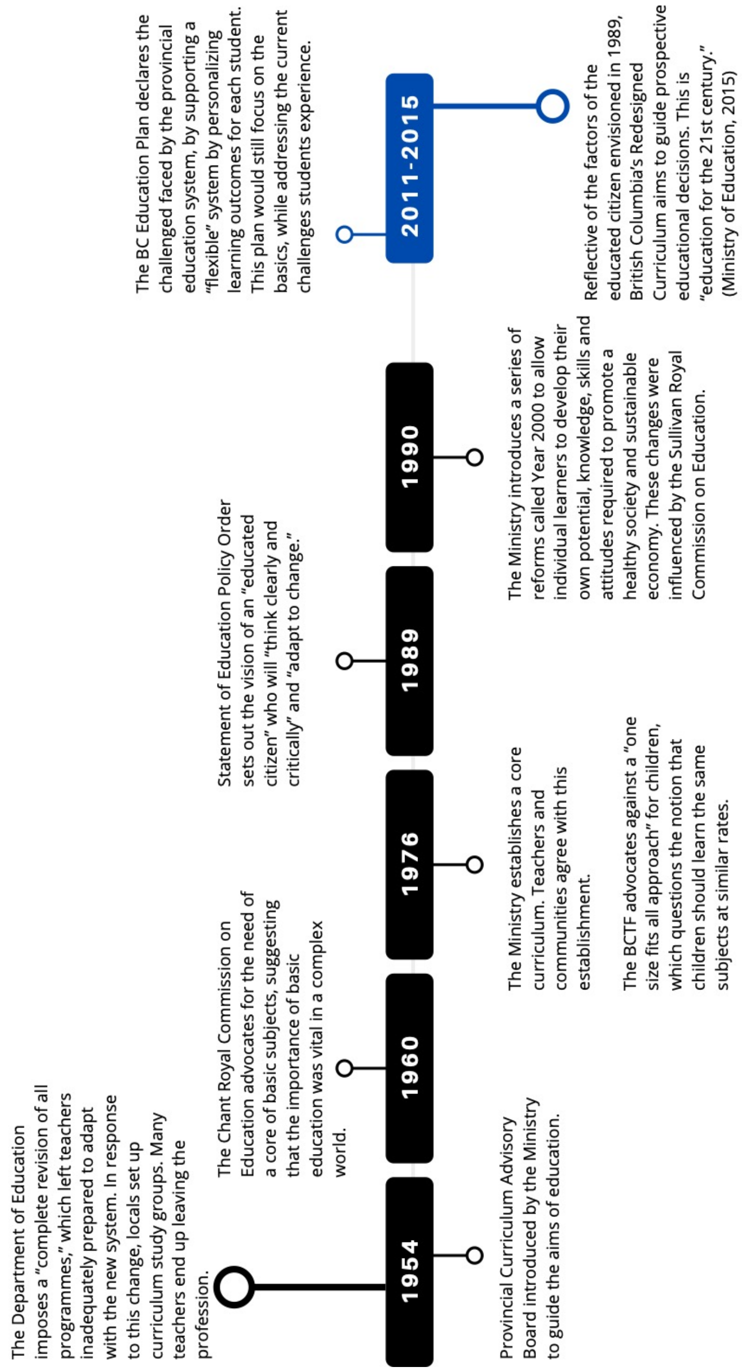
3. British Columbia Teachers' Federation. (2021, January). Class size and composition. https://bctf.ca/issuesineducation.aspx?id=5530
4. British Columbia Teachers' Federation. (2019). Public Schools, Class Sizes, and Composition. https://bctf.ca/uploadedFiles/Public/Publications/FactSheets/PublicSchoolsCSC.pdf
5. Government of British Columbia. (2021, January). B.C.'s curriculum – core competencies. https://curriculum.gov.bc.ca/competencies

Figure B4

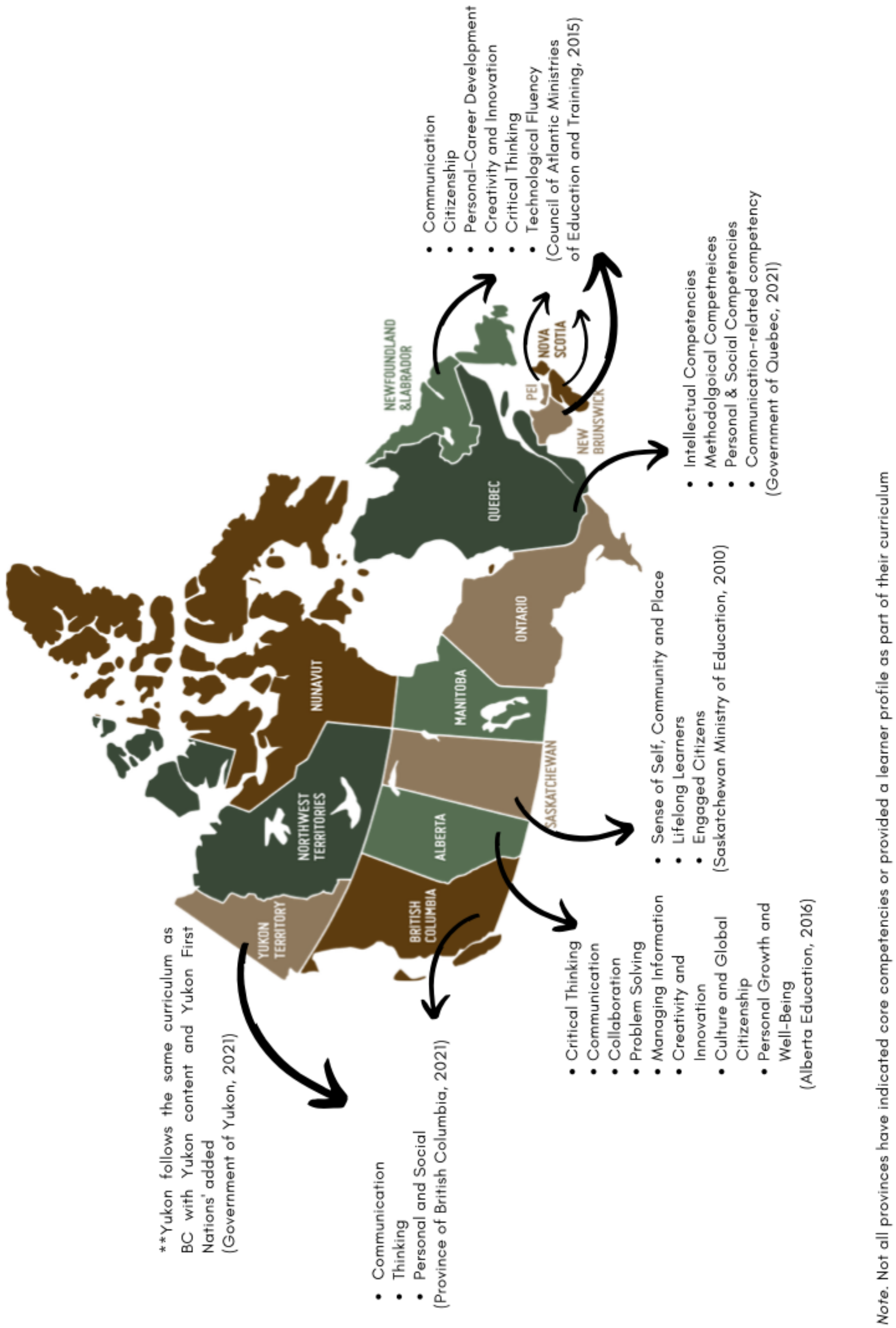
My Timeline of BC's Curriculum



Note. Figure was created by the BCTF (retrieved 2020) as a way of representing the changes that have occurred in the BC Curriculum.

Figure B5

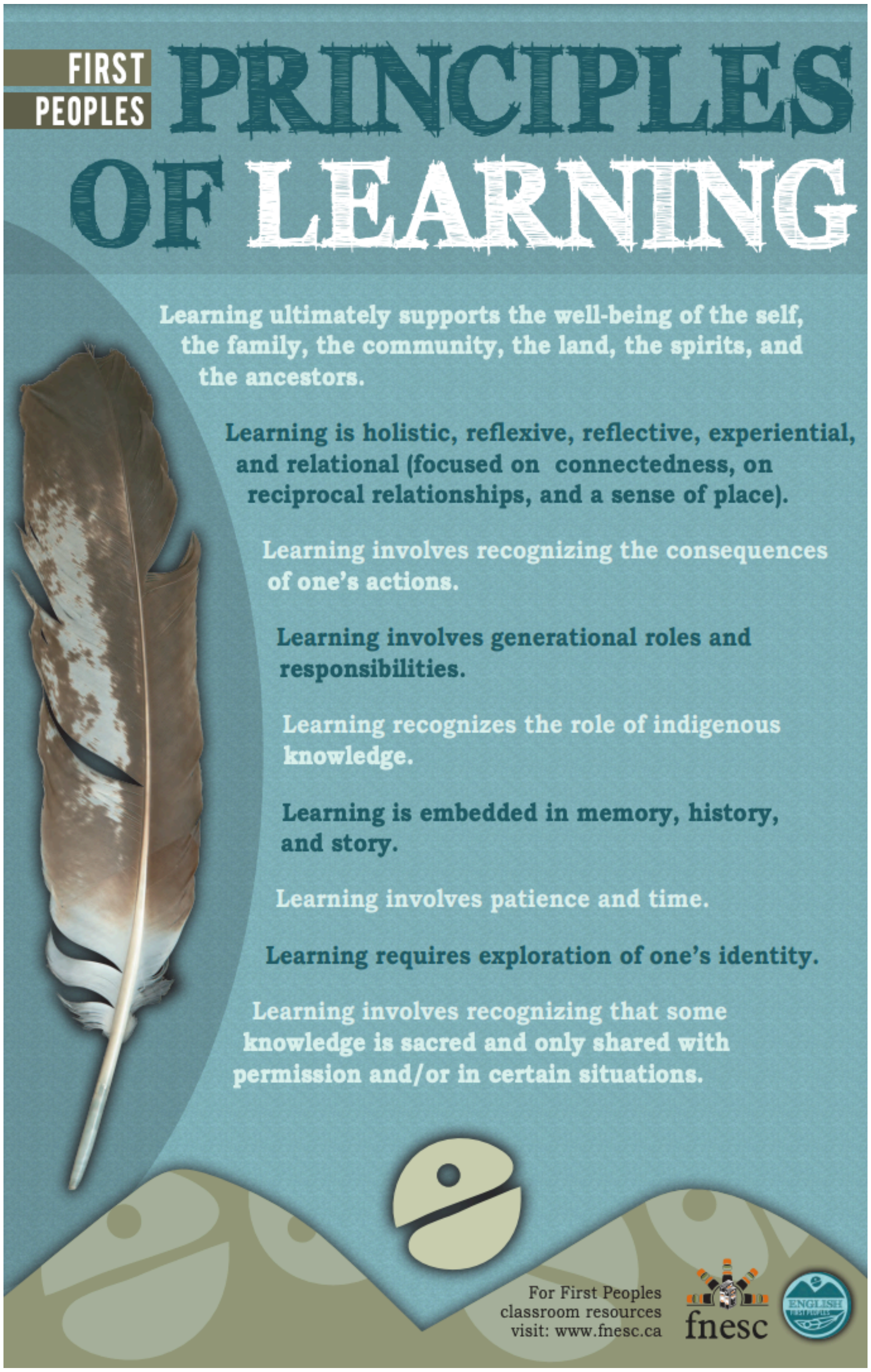
My Overview of Core Competencies in Canada



Note. All information listed is a general representation (K-12) as stated on provincial website.

Figure B6

First Peoples Principles of Learning



Note. This figure was created by the First Nations Education Steering Committee and has not been adapted or changed in any way.

Table B1

Differences Between The 2007 and 2020 Island Numeracy Assessments for Grade three Students (INA3+)

	2007 Edition	2020 Edition
	Number of Questions	
Patterning	4	13
Number Sense	20	19
Computational Fluency	13	16
Reflection	1	3

Note. Both editions (2007 & 20020) of the INA contain the same topics.

Table B2

Ability Grouping by Levels of Streaming and Tracking According to PISA

Note. Countries are in order according to PISA ranking.

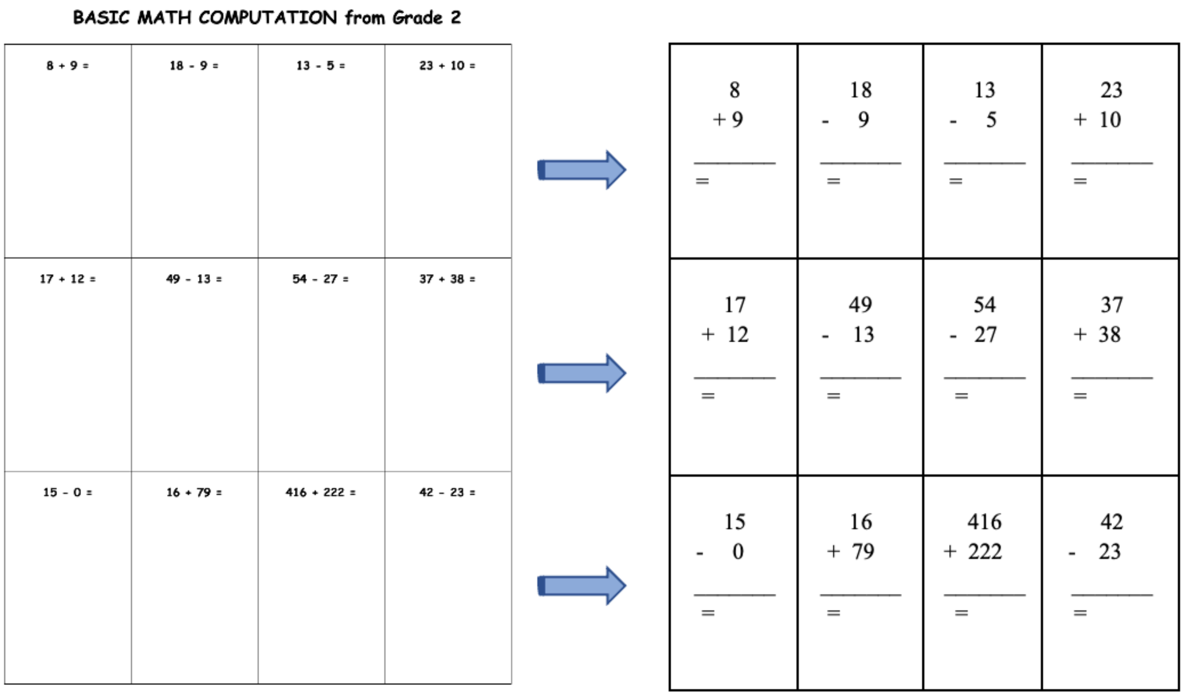
Ability Grouping by levels of streaming (*i.e.*, grouping within schools) and tracking (*i.e.*, grouping within classrooms; Chiu et al., 2017)

Country		PISA 2018 Mathematics Results
Singapore	Over 80% of students are grouped by ability in mathematics classes as confirmed by school principals (Organisation for Economic Co-operation and Development (OECD), 2013)	569
Japan	Schools that did not practice ability grouping in classes for 15-year-olds dropped by over 15 percent (OECD, 2013)	527
South Korea	Ability grouping has become more common than for some or no classes; schools that did not practice ability grouping in classes for 15-year-olds dropped by over 15 percent (OECD, 2013)	526
Poland	Ability grouping has become less common, with a 24 percent drop in the use of no ability grouping from 2003 to 2012 (OECD, 2013)	516
Canada	"The gap in academic achievement between streamed students from high and low socio-economic backgrounds increases with age, with the achievement gap in Mathematics doubling from age 7–11 to 12–15" (Caro, 2009)	512
Denmark	Ability grouping has become more common; schools no longer have some or all classes that to not practice ability grouping; schools that did not practice ability grouping in classes for 15-year-olds dropped by over 15 percent (OECD, 2013)	509
Finland	Streaming and tracking is performed until the end of Grade 9 (Aedo, et al. 2017)	507
Sweden	43% of secondary schools use ability grouping (Ramberg, 2015)	502
United Kingdom	71% of secondary students are taught using streams for mathematics (Francis et al., 2017); over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	502
Germany	The number of students who joined schools that practiced ability grouping in some or all classes increased between 2003 and 2012; schools that did not practice ability grouping in classes for 15-year-olds dropped by over 20 percent (OECD, 2013)	500
Ireland	Over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	500
New Zealand	Over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	494
Australia	98 percent of Australia's schools use some form of streaming (OECD, 2012); over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	491
Israel	Over 80% of students are grouped by ability in mathematics classes as confirmed my school principals (OECD, 2013)	463

Appendix C - Chapter 3: Research Design and Methods

Figure C1

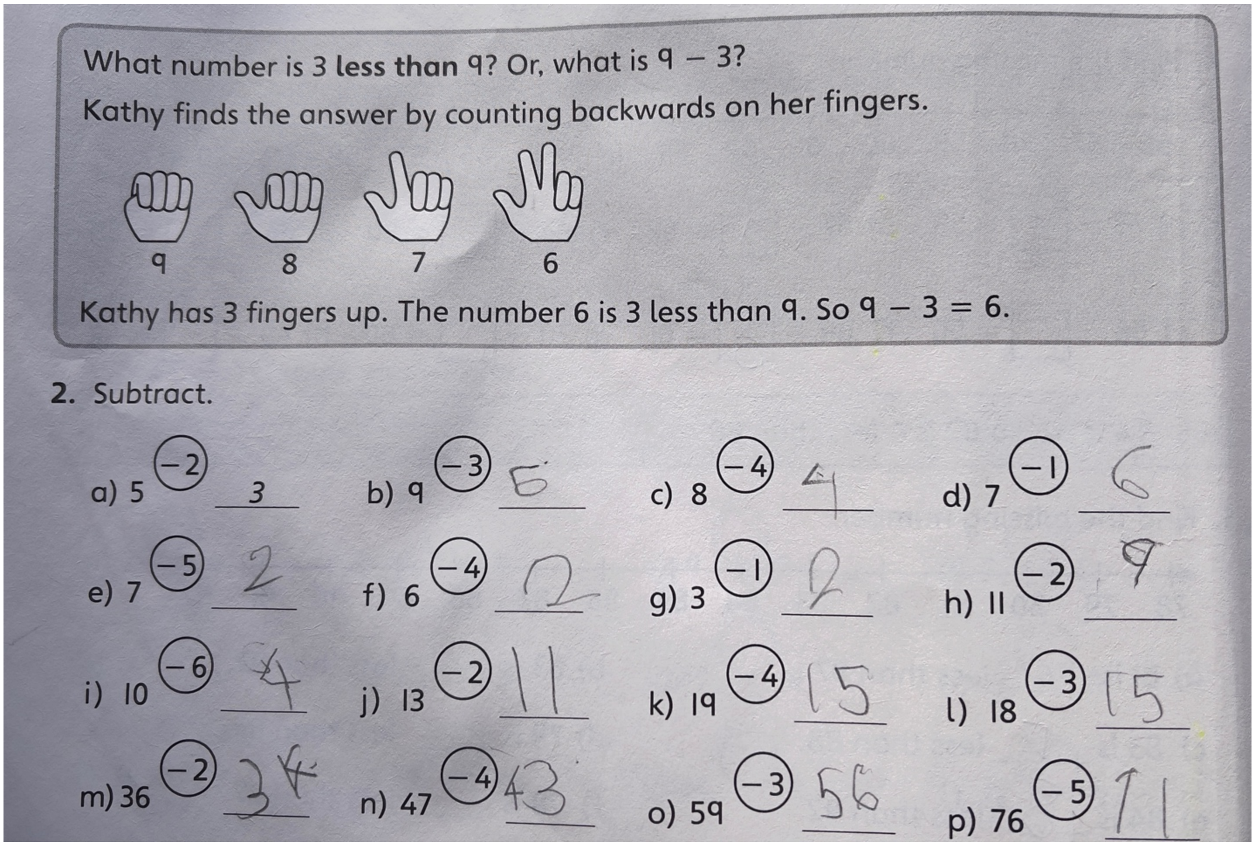
Sample of Math Computations of The Island Numeracy Assessment (INA)



Note. Formatting was adapted from the INA's basic math computation (Island Numeracy, 2007) from horizontal to vertical (right).

Figure C2

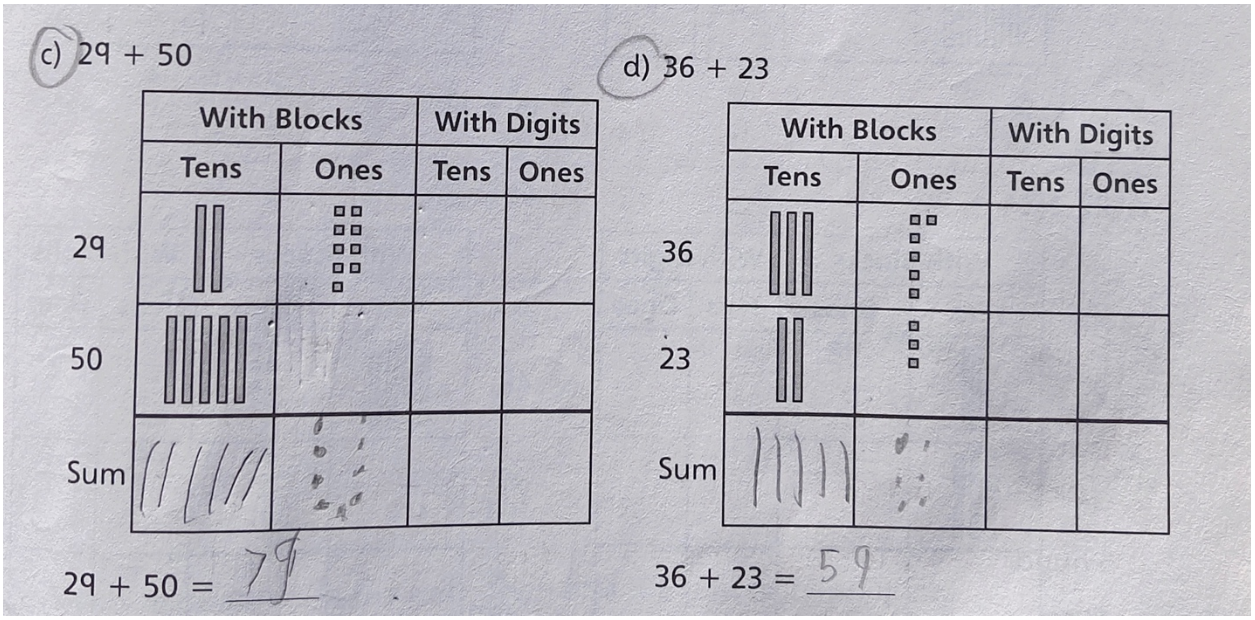
Student Work Sample – Subtraction



Note. Format for teaching subtraction according to JUMP Math.

Figure C3

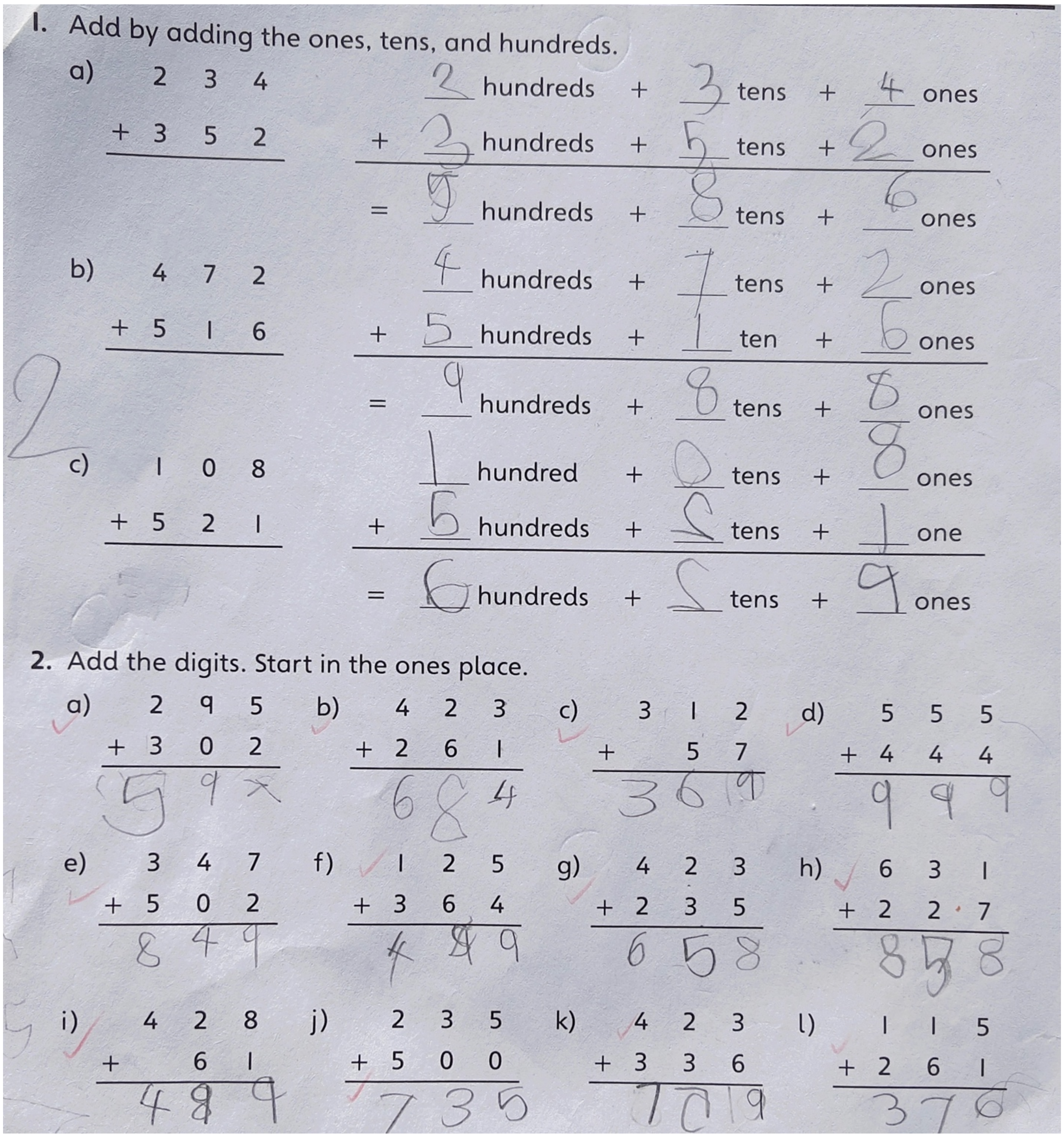
Student Work Sample – Number Sense



Note. Teaching addition with base ten blocks.

Figure C4

Student Work Sample – Addition



Note. Next step in teaching addition according to JUMP Math.

Table C1

Distribution of Demographic Characteristics Across Sex and Age According to Birth Month

	N = 71	%
Sex		
Male	39	54.9
Female	32	45.1
Age		
January	6	8.45
February	4	5.63
March	11	15.49
April	3	4.23
May	6	8.45
June	4	5.63
July	6	8.45
August	2	2.82
September	5	7.04
October	11	15.49
November	5	7.04
December	4	5.63
Born in 2009/2010	4	5.63

Table C2

Distribution of Demographic Characteristics Across Ethnicity, Language and Family Structure

	N = 71	%
Ethnicity		
Caucasian / White	16	19.72
East Asian (Chinese, Japanese, Korean, Taiwanese)	40	40.8
African / Black	3	4.23
South Asian (e.g., Indian, Pakistani, Sri Lankan)	2	2.82
Other	12	16.9
Mother tongue		
English	41	57.7
Chinese	5	7.04
Korean	5	7.04
Mandarin	15	21.1
Other	5	7.04
Second Language Spoken at Home		
No	40	56.3
Yes	31	43.7
Number of Children in The Family		
One	5	7.04
Two	43	60.6
Three	16	22.5
Four	6	8.45
Six	1	1.41
Position of the Child		

Oldest	24	33.8
Middle	8	11.3
Youngest	34	47.9
No siblings	5	7.04
Marital Status of Parents		
Married	64	90.1
Divorced	7	9.86
Education of Mother		
High School	3	4.23
Certificate Program or Some Post-Secondary	57	80.3
Master's Degree	8	11.3
Did Not State	3	4.23
Education of Father		
High School	7	9.86
Certificate Program or Some Post-Secondary	41	57.7
Master's Degree	13	18.3
PhD	5	7.04
Did Not State	5	7.04
Individualized Education Plan (IEP) or Learning Plan (LP)		
None	67	94.4
Learning Plan	3	4.23
IEP	1	1.41

Appendix D - Chapter 4: Data Analysis

Table D1

Distribution of Sex Across Groups

	Male	Female
Peer-Based Group	13	12
Academic-Based Group	23	11
Control Group	9	14
Total	35	37

Table D2

Distribution of Students by Birth Month and Group

	Birth Month	Birthdate Group	Oldest to Youngest by Group
	Number of Students		
Born in 2009/2010	3	3	1
January	6	24	2
February	4		
March	11		
April	3		
May	6	18	3
June	4		
July	6		
August	2		
September	5	25	4
October	11		
November	5		
December	4		

Note. 1 = oldest; 4 = youngest

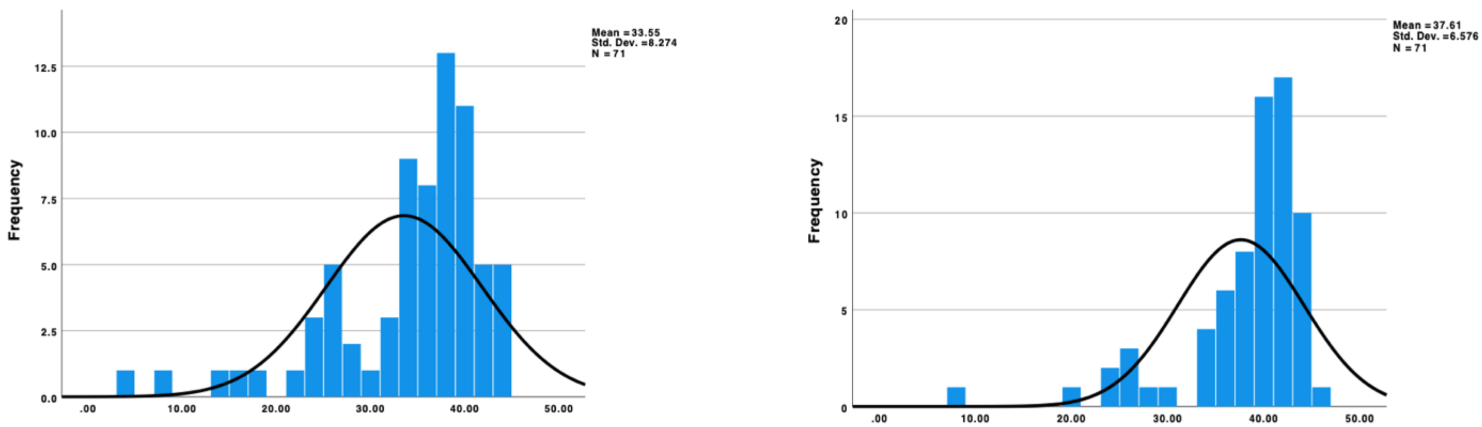
Table D3

Skewness and Kurtosis for Pre and Post Test

	Skewness	Kurtosis	Outliers
Pre-Test			
Peer-Based Group	-1.81	3.11	3.00
Academic Based Group	-0.54	-0.51	0.00
Control Group	-2.01	4.41	1.00
Post-Test			
Peer-Based Group	-3.33	-1.83	-1.83
Academic Based Group	3.18	2.36	2.36
Control Group	0.00	1.00	0.00

Figure D1

Histogram of Pre-Test and Post-Test Scores



Note. Pre-test scores (left side); Post-test scores (right side)

Figure D2

Comparison of academic performance across groups

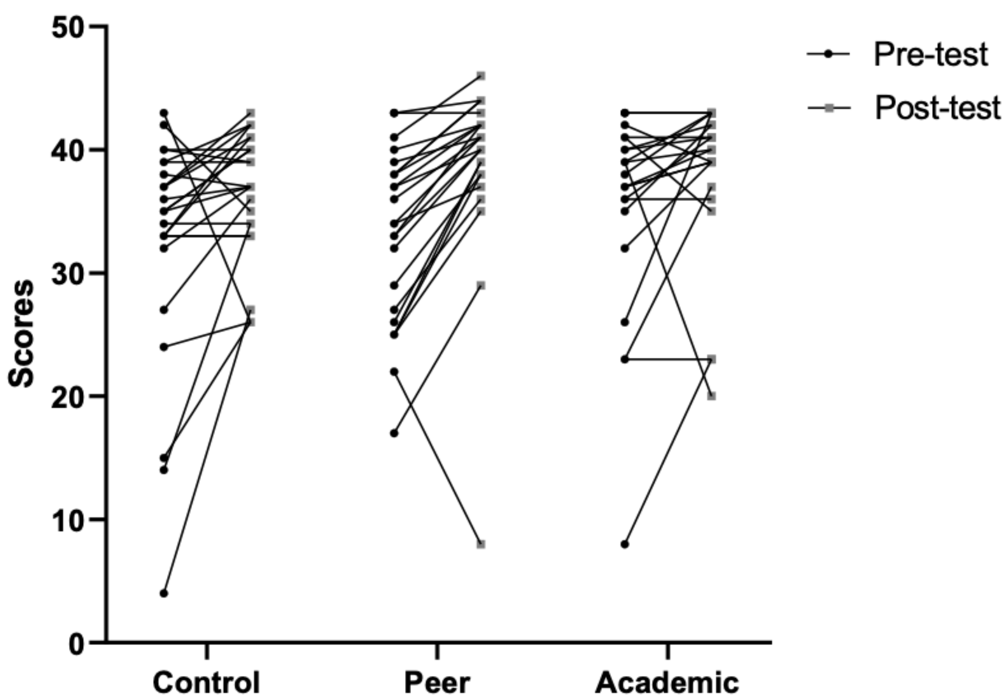


Figure D3

Student Sample from INA

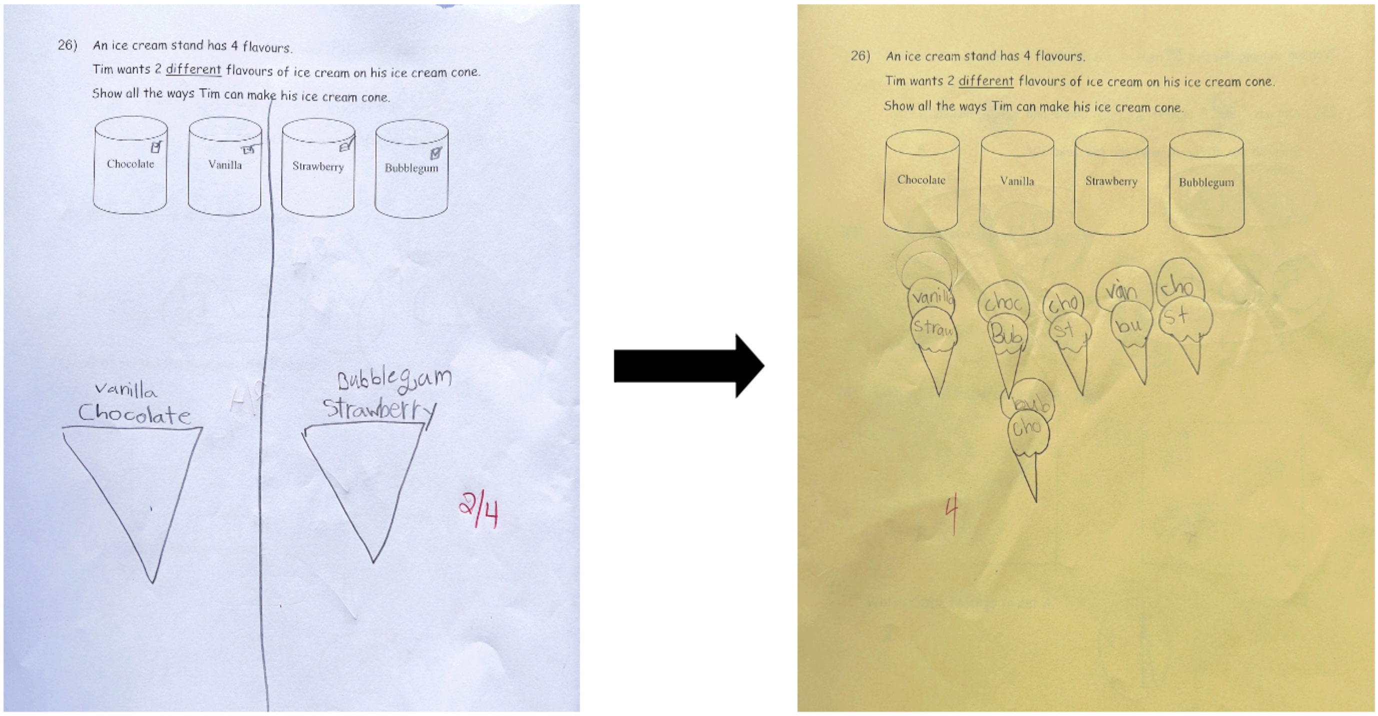


Figure D4

Mean comparison of post-test scores across sex

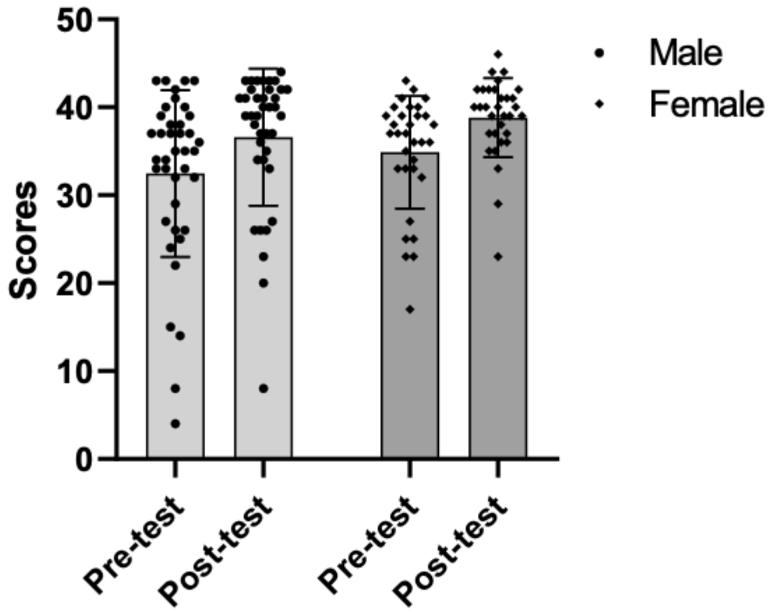


Figure D5

Mean scores across birthdates

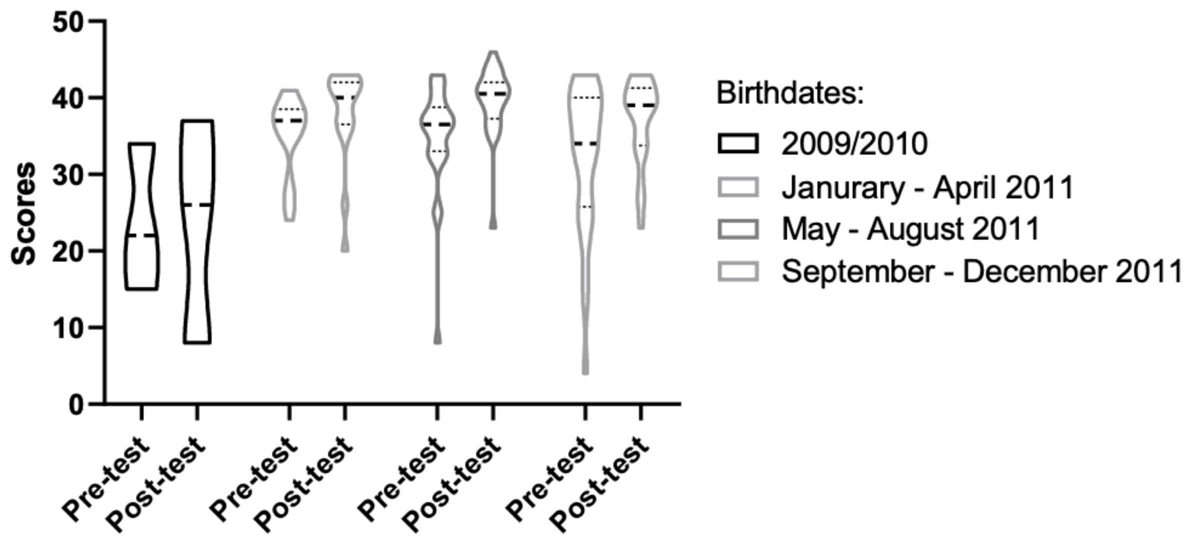


Figure D6

Comparison of birthdate scores for Academic-Based Group

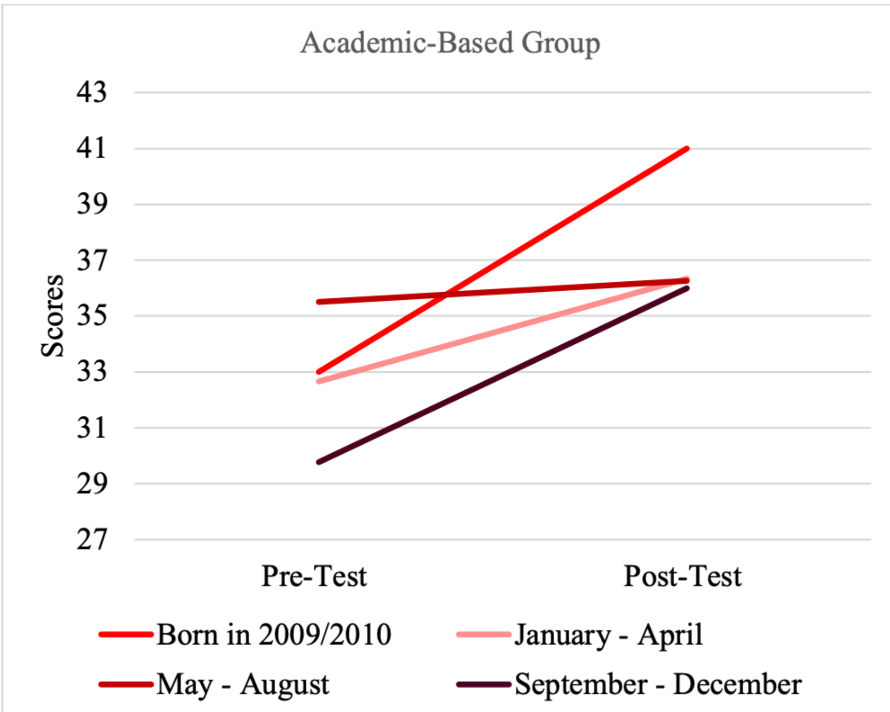


Figure D7

Comparison of birthdate scores for Peer-Based Group

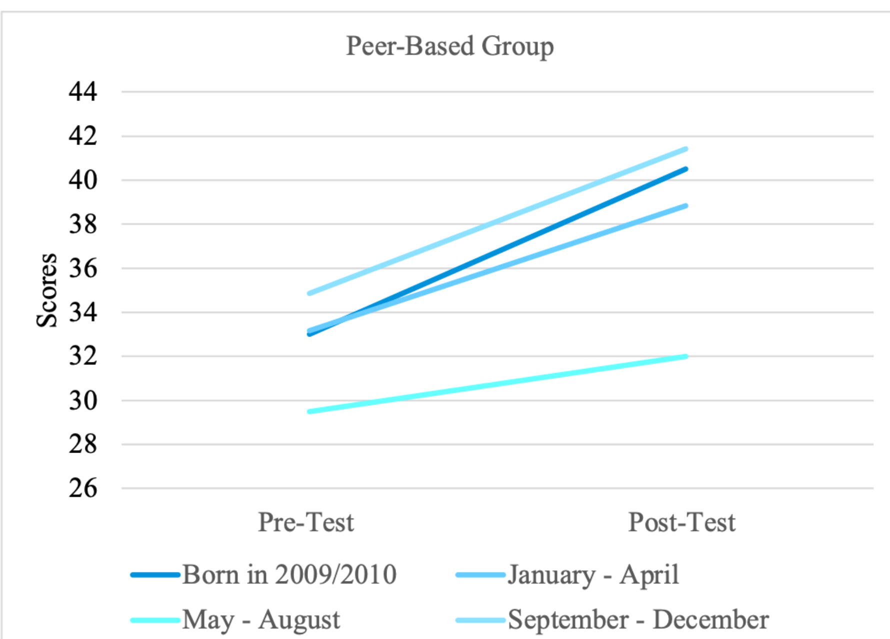
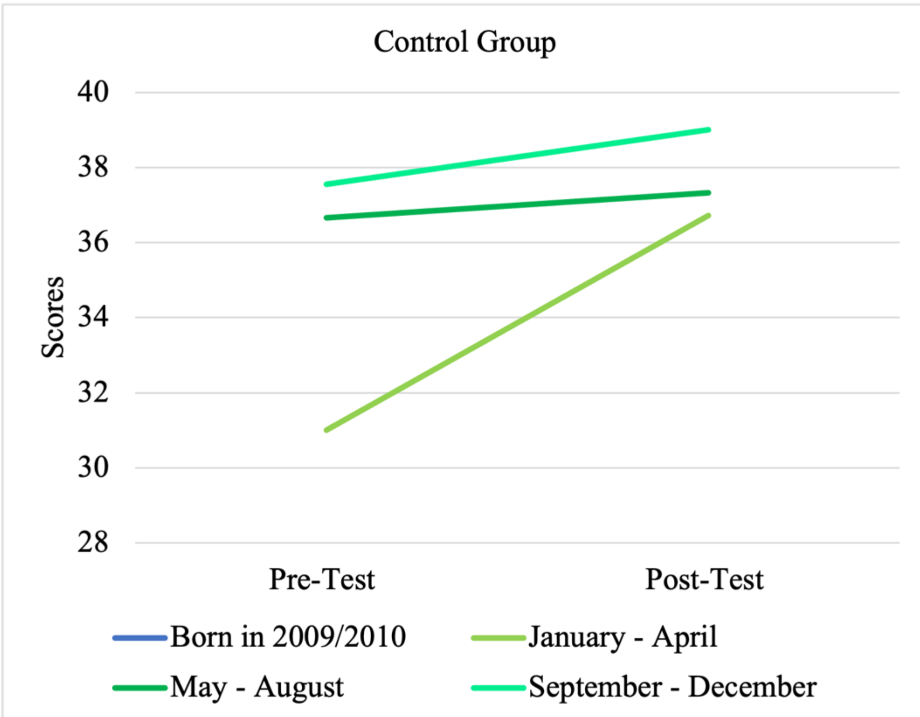


Figure D8

Comparison of birthdate scores for Control Group



Note. There were no students born in 2009/2010 in the control group

Appendix E: Example of Consent Letter Sent to Parents

September 12, 2019

Dear Mrs/Mr ______________________

RE: PARENT/GUARDIAN - REQUEST FOR PERMISSION TO CONDUCT A STUDY

My name is Rachel Poon. I am currently a student at Trinity Western University; doing my Masters degree. The title of my study is "Math Groups: Skill Based or Peer Based." Part of this study includes a mathematical competence test to assess each individual in order to understand the mathematical development of a 7-8 year old child.
I hereby request your permission to conduct this test on your daughter/son, ____________________________.

These tests will be conducted at XXX and will take place during September 2019 and February 2020. Students will complete a diagnostic math assessment in their individual classrooms during this time.

Your child's participation in this research process is voluntary and she/he may choose to withdraw from this process at any time. If she/he chooses to not participate or withdraw from the research process, there will be no penalty.

For the purposes of anonymity and confidentiality the names of the school and your child will not be mentioned throughout the data and findings of the case study. Pseudonyms will be used in the writing of the final assignment. The collected data will be for this case study only, and apart from myself, only the lecturer will have access to the data. She will treat this information as highly confidential. Should you so require, I will share the findings with you once my assignment has been completed.

Should you have any questions or enquiries about this case study, please do not hesitate to contact me or my supervisor.

Thanking you in advance

Yours sincerely,
Rachel Poon

Appendix F: Ethics

Figure F1

Letter of confirmation from TWU REB

Dear Rachel,
Thank you for your email clarifying questions regarding your research project "Math groups: Skill based or peer based".
The Board accepts your clarifications given to our questions. We are not able to give full approval of your study until we receive confirmation from [redacted] giving approval to this study.
Please submit this document so we can move forward to approval.
Best wishes,
Landa
LANDA TERBLANCHE PhD RN
Assistant Dean & Associate Professor | School of Nursing
Co-Chair HREB

Figure F2

Letter of approval from School X

To Whom It May Concern:

Rachel Poon is currently employed at [redacted] as a grade 3 teacher and is granted permission to conduct her Master's Research Project at the grade 3 level as outlined below.

Research Project Name: Math Groups: Skill Based or Peer Based

Description / Rationale: In British Columbia, students are often put into different streams for both mathematics and science programs when as they enter High School. In other countries, the age at which streaming occurs can begin younger. Although streaming does not occur at a young age for all countries, many teachers do put students into groups as a way of targeting specific needs. The goal of this study is to gain a better understanding to determine if student grouping (skills based vs peer based) is effective and which type of grouping students might benefit from most.

If you have any further questions or concerns, please do not hesitate to contact me.

Sincerely,

Note. A full copy of my REB application is available upon request

Appendix G: Island Numeracy Assessment

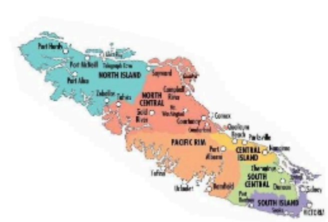
End of Grade 2 I.R.P.

Beginning of Grade 3

Diagnostic Math Assessment

Last updated: December 10, 2007

WNCP
Edition



Vancouver IslandNet

Note. A complete copy of the 2007 INA is available upon request. The 2020 INA can be found at: http://www.islandnumeracy.ca/index.php/ina-home/grade-3-assessment-materials/.

Appendix H: Original SPSS Data Output

Figure H1

Sex

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male	39	54.9	54.9	54.9
	Female	32	45.1	45.1	100.0
	Total	71	100.0	100.0	

Figure H2

Group

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Peer Based	24	33.8	33.8	33.8
	Academic Based	25	35.2	35.2	69.0
	Control	22	31.0	31.0	100.0
	Total	71	100.0	100.0	

Figure H3

BDay (According to Age Group)

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Born 2010 or 2009	3	4.2	4.2	4.2
	May –August (Middle)	18	25.4	25.4	29.6
	January–April (Oldest)	25	35.2	35.2	64.8
	September–December (Youngest)	25	35.2	35.2	100.0
	Total	71	100.0	100.0	

Figure H4

Ethnicity

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Caucasian/White	14	19.7	19.7	19.7
	East Asian (Chinese, Japanese, Korean, Taiwanese)	40	56.3	56.3	76.1
	African	3	4.2	4.2	80.3
	South Asian (Indian, Pakistani, Sri Lankan)	2	2.8	2.8	83.1
	Other	12	16.9	16.9	100.0
	Total	71	100.0	100.0	

Figure H5

Mother Tongue

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	English	41	57.7	57.7	57.7
	Chinese	5	7.0	7.0	64.8
	Korean	5	7.0	7.0	71.8
	Mandarin	4	5.6	5.6	77.5
	Other	16	22.5	22.5	100.0
	Total	71	100.0	100.0	

Figure H6

Number of Children in Family

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1.00	5	7.0	7.0	7.0
	2.00	43	60.6	60.6	67.6
	3.00	16	22.5	22.5	90.1
	4.00	6	8.5	8.5	98.6
	6.00	1	1.4	1.4	100.0
	Total	71	100.0	100.0	

Figure H7

Position of Child in Family

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Oldest	24	33.8	33.8	33.8
	Middle	8	11.3	11.3	45.1
	Youngest	34	47.9	47.9	93.0
	No Siblings	5	7.0	7.0	100.0
	Total	71	100.0	100.0	

Figure H8

Education of Mother

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	High School	3	4.2	4.2	4.2
	Certificate Program or Some Post Secondary	57	80.3	80.3	84.5
	Masters Degree	8	11.3	11.3	95.8
	Did Not State	3	4.2	4.2	100.0
	Total	71	100.0	100.0	

Education of Father

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	High School	7	9.9	9.9	9.9
	Certificate Program or Some Post Secondary	41	57.7	57.7	67.6
	Masters Degree	13	18.3	18.3	85.9
	PHD	5	7.0	7.0	93.0
	Did Not State	5	7.0	7.0	100.0
	Total	71	100.0	100.0	

Figure H9

IEP/Learnig Plan

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	None	67	94.4	94.4	94.4
	Learning Plan	3	4.2	4.2	98.6
	IEP	1	1.4	1.4	100.0
	Total	71	100.0	100.0	

Figure H10

Marital Status of Parents

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Married	64	90.1	90.1	90.1
	Divorced	7	9.9	9.9	100.0
	Total	71	100.0	100.0	

Figure H11

PreTest (all)

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	4.00	1	1.4	1.4	1.4
	8.00	1	1.4	1.4	2.8
	14.00	1	1.4	1.4	4.2
	15.00	1	1.4	1.4	5.6
	17.00	1	1.4	1.4	7.0
	22.00	1	1.4	1.4	8.5
	23.00	2	2.8	2.8	11.3
	24.00	1	1.4	1.4	12.7
	25.00	3	4.2	4.2	16.9
	26.00	2	2.8	2.8	19.7
	27.00	2	2.8	2.8	22.5
	29.00	1	1.4	1.4	23.9
	32.00	3	4.2	4.2	28.2
	33.00	6	8.5	8.5	36.6
	34.00	3	4.2	4.2	40.8
	35.00	4	5.6	5.6	46.5
	36.00	4	5.6	5.6	52.1
	37.00	8	11.3	11.3	63.4
	38.00	5	7.0	7.0	70.4
	39.00	6	8.5	8.5	78.9
	40.00	5	7.0	7.0	85.9
	41.00	3	4.2	4.2	90.1
	42.00	2	2.8	2.8	93.0
	43.00	5	7.0	7.0	100.0
	Total	71	100.0	100.0	

Figure H12

PostTest Scores

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	8.00	1	1.4	1.4	1.4
	20.00	1	1.4	1.4	2.8
	23.00	2	2.8	2.8	5.6
	26.00	3	4.2	4.2	9.9
	27.00	1	1.4	1.4	11.3
	29.00	1	1.4	1.4	12.7
	33.00	2	2.8	2.8	15.5
	34.00	2	2.8	2.8	18.3
	35.00	3	4.2	4.2	22.5
	36.00	3	4.2	4.2	26.8
	37.00	6	8.5	8.5	35.2
	38.00	2	2.8	2.8	38.0
	39.00	8	11.3	11.3	49.3
	40.00	8	11.3	11.3	60.6
	41.00	8	11.3	11.3	71.8
	42.00	9	12.7	12.7	84.5
	43.00	7	9.9	9.9	94.4
	44.00	3	4.2	4.2	98.6
	46.00	1	1.4	1.4	100.0
	Total	71	100.0	100.0	

Figure H13

Descriptives

			Statistic	Std. Error
PreTest (all)	Mean		33.5493	.98189
	95% Confidence Interval for Mean	Lower Bound	31.5910	
		Upper Bound	35.5076	
	5% Trimmed Mean		34.3521	
	Median		36.0000	
	Variance		68.451	
	Std. Deviation		8.27352	
	Minimum		4.00	
	Maximum		43.00	
	Range		39.00	
	Interquartile Range		7.00	
	Skewness		-1.537	.285
	Kurtosis		2.454	.563
testPost(all)	Mean		37.6056	.78041
	95% Confidence Interval for Mean	Lower Bound	36.0491	
		Upper Bound	39.1621	
	5% Trimmed Mean		38.3122	
	Median		40.0000	
	Variance		43.242	
	Std. Deviation		6.57588	
	Minimum		8.00	
	Maximum		46.00	
	Range		38.00	
	Interquartile Range		6.00	
	Skewness		-2.111	.285
	Kurtosis		5.606	.563

Figure H14

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
PreTest (all)	.192	71	<.001	.854	71	<.001
testPost(all)	.204	71	<.001	.789	71	<.001

a. Lilliefors Significance Correction

Figure H15

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Gender	.365	71	<.001	.633	71	<.001
BDay Age Groups	.219	71	<.001	.841	71	<.001

a. Lilliefors Significance Correction

Figure H16

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Treatment	Sphericity Assumed	78944.454	1	78944.454	1837.568	<.001	.963
	Greenhouse-Geisser	78944.454	1.000	78944.454	1837.568	<.001	.963
	Huynh-Feldt	78944.454	1.000	78944.454	1837.568	<.001	.963
	Lower-bound	78944.454	1.000	78944.454	1837.568	<.001	.963
Error(Treatment)	Sphericity Assumed	3007.296	70	42.961			
	Greenhouse-Geisser	3007.296	70.000	42.961			
	Huynh-Feldt	3007.296	70.000	42.961			
	Lower-bound	3007.296	70.000	42.961			
Time	Sphericity Assumed	560.567	1	560.567	43.350	<.001	.382
	Greenhouse-Geisser	560.567	1.000	560.567	43.350	<.001	.382
	Huynh-Feldt	560.567	1.000	560.567	43.350	<.001	.382
	Lower-bound	560.567	1.000	560.567	43.350	<.001	.382
Error(Time)	Sphericity Assumed	905.183	70	12.931			
	Greenhouse-Geisser	905.183	70.000	12.931			
	Huynh-Feldt	905.183	70.000	12.931			
	Lower-bound	905.183	70.000	12.931			
Treatment * Time	Sphericity Assumed	110.313	1	110.313	8.771	.004	.111
	Greenhouse-Geisser	110.313	1.000	110.313	8.771	.004	.111
	Huynh-Feldt	110.313	1.000	110.313	8.771	.004	.111
	Lower-bound	110.313	1.000	110.313	8.771	.004	.111
Error(Treatment*Time)	Sphericity Assumed	880.437	70	12.578			
	Greenhouse-Geisser	880.437	70.000	12.578			
	Huynh-Feldt	880.437	70.000	12.578			
	Lower-bound	880.437	70.000	12.578			

Figure H17

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Time	Sphericity Assumed	355.058	1	355.058	13.810	<.001	.187
	Greenhouse-Geisser	355.058	1.000	355.058	13.810	<.001	.187
	Huynh-Feldt	355.058	1.000	355.058	13.810	<.001	.187
	Lower-bound	355.058	1.000	355.058	13.810	<.001	.187
Time * Subjects	Sphericity Assumed	20.785	2	10.392	.404	.669	.013
	Greenhouse-Geisser	20.785	2.000	10.392	.404	.669	.013
	Huynh-Feldt	20.785	2.000	10.392	.404	.669	.013
	Lower-bound	20.785	2.000	10.392	.404	.669	.013
Time * BDayGroups	Sphericity Assumed	91.914	3	30.638	1.192	.321	.056
	Greenhouse-Geisser	91.914	3.000	30.638	1.192	.321	.056
	Huynh-Feldt	91.914	3.000	30.638	1.192	.321	.056
	Lower-bound	91.914	3.000	30.638	1.192	.321	.056
Time * Subjects * BDayGroups	Sphericity Assumed	53.640	5	10.728	.417	.835	.034
	Greenhouse-Geisser	53.640	5.000	10.728	.417	.835	.034
	Huynh-Feldt	53.640	5.000	10.728	.417	.835	.034
	Lower-bound	53.640	5.000	10.728	.417	.835	.034
Error(Time)	Sphericity Assumed	1542.627	60	25.710			
	Greenhouse-Geisser	1542.627	60.000	25.710			
	Huynh-Feldt	1542.627	60.000	25.710			
	Lower-bound	1542.627	60.000	25.710			

Figure H18

BDay Age Groups

Multiple Comparisons

Measure: MEASURE_1
Tukey HSD

(I) BDay Age Groups	(J) BDay Age Groups	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Born 2010 or 2009	May -August (Middle)	1.7222	4.19831	.976	-9.3719	12.8163
	January-April (Oldest)	1.7933	4.11349	.972	-9.0766	12.6633
	September-December (Youngest)	.5333	4.11349	.999	-10.3366	11.4033
May -August (Middle)	Born 2010 or 2009	-1.7222	4.19831	.976	-12.8163	9.3719
	January-April (Oldest)	.0711	2.08108	1.000	-5.4282	5.5704
	September-December (Youngest)	-1.1889	2.08108	.940	-6.6882	4.3104
January-April (Oldest)	Born 2010 or 2009	-1.7933	4.11349	.972	-12.6633	9.0766
	May -August (Middle)	-.0711	2.08108	1.000	-5.5704	5.4282
	September-December (Youngest)	-1.2600	1.90417	.911	-6.2918	3.7718
September-December (Youngest)	Born 2010 or 2009	-.5333	4.11349	.999	-11.4033	10.3366
	May -August (Middle)	1.1889	2.08108	.940	-4.3104	6.6882
	January-April (Oldest)	1.2600	1.90417	.911	-3.7718	6.2918

Based on observed means.
The error term is Mean Square(Error) = 45.323.

Appendix I: Editing

Figure I1
Letter from Editor

To whom it may concern,

I am an editor/grant writer with a background in teaching and tutoring university students in the English language. Please find a list below of the editing that I was able to assist Rachel Poon with her manuscript:

Group Structuring in Elementary School Mathematics:
Peer-Based or Academic-Based

- Spelling
- Punctuation
- Sentence structure
- APA formatting: in-text citations and reference list
- Grammar
- Graphs and pictures
- Table of contents
- Indentations and margins
- Headings

If you further questions, I can be contacted at: slkostamo@gmail.com.

Sincerely,

Shauna Kostamo, Editor